Quantum Chaos: An introduction via chains of interacting spins 1/2

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What is quantum chaos?

- Classical chaos
  - Hypersensitivity to initial conditions
  - Dynamical billiard: billiard table with no friction and elastic collisions.
  - Depending on the shape: chaos
  - In phase space, the trajectories of two particles with very close initial conditions will diverge exponentially in time (rate=Lyapunov exponent)

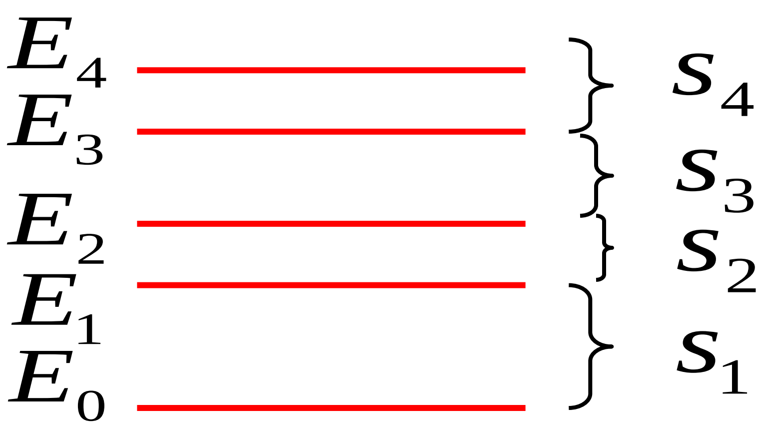
What is quantum chaos?

- Classical chaos
  - Hypersensitivity to initial conditions
- Quantum chaos
  - Cannot use hypersensitivity due to Heisenberg's Uncertainty Principle
  - Classical systems are a limit of quantum systems
  - Quantum billiards: distribution of neighboring energy levels depends on the billiard's classical counterpart.

Level spacing distribution

Histogram of the spacings between neighboring energy levels

Energy levels   Energy spacings



Level spacing distribution

- When classical billiard was chaotic, the energy levels of the quantum billiard are highly correlated and repel each other
- The distribution is given by the Wigner-Dyson distribution, $P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4}$
- In an integrable (non-chaotic) system, the energy levels may cross
- The distribution is Poissonian, $P(s) = e^{-s}$

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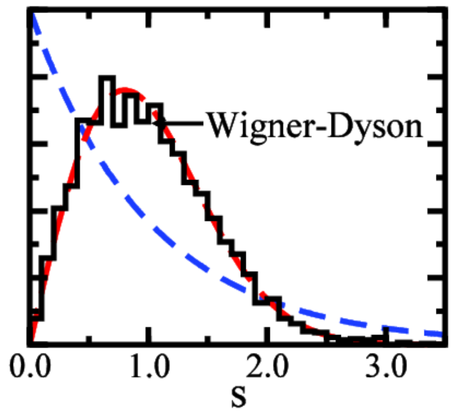


The system

- We study a 1D system of spins-1/2 with L sites
- Each site contains either a spin up or a spin down
- We use a chain with L/3 spins up (excitations)

The Hamiltonian

- Our system is described by the Hamiltonian

$$H = H_z + H_{NN} \qquad H_z = \sum_{n=1}^{L} \varepsilon_n S_n^z$$

Clean $\varepsilon_n = \varepsilon$

Defect -- site with different Zeeman splitting: $\begin{cases} \varepsilon_{n \neq m} = \varepsilon \\ \varepsilon_m = \varepsilon + d_m \end{cases}$

$$H_{NN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]$$

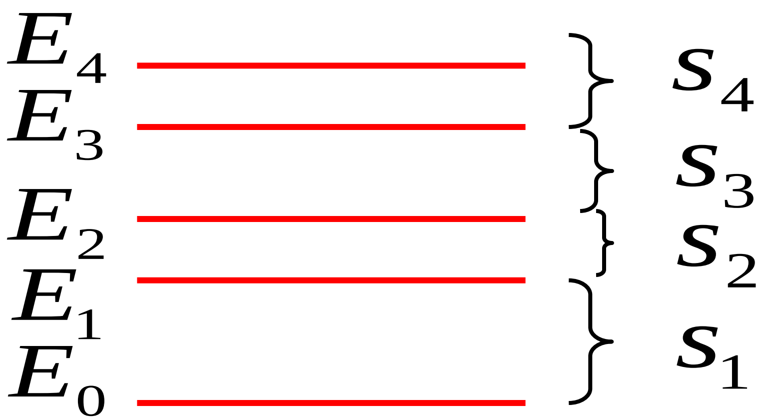
$$H_{zz} = \sum_{n=1}^{L-1} J_z S_n^z S_{n+1}^z \qquad H_{XY} = \sum_{n=1}^{L-1} \left[J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]$$

Ising Interaction — Flip-Flop Term

Level spacing distribution

Histogram of the spacings between neighboring energy levels

Energy levels　　　　　Energy spacings

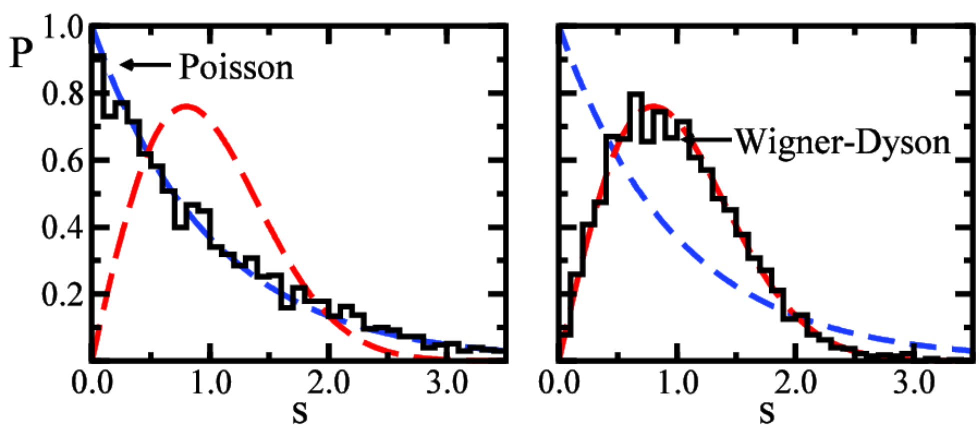


Level spacing distribution

- In chaotic systems, the energy levels are highly correlated and repel each other
- The distribution is given by the Wigner-Dyson distribution, $P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4}$
- In an integrable (non-chaotic) system, the energy levels may cross
- The distribution is Poissonian, $P(s) = e^{-s}$

Level spacing distribution

- When the defect is placed on site 1, we obtain the Poissonian distribution, corresponding to an integrable system
- When the defect is placed on site L/2, we obtain the Wigner-Dyson distribution, corresponding to a chaotic system



L = 15
5 spins up
Jz = .5 J
Eps1, Eps L/2 = .5 J

Number of Principal Components

- NPC is a measure of the delocalization of eigenstates
  - It gives the number of basis vectors Φ which contribute to each eigenstate

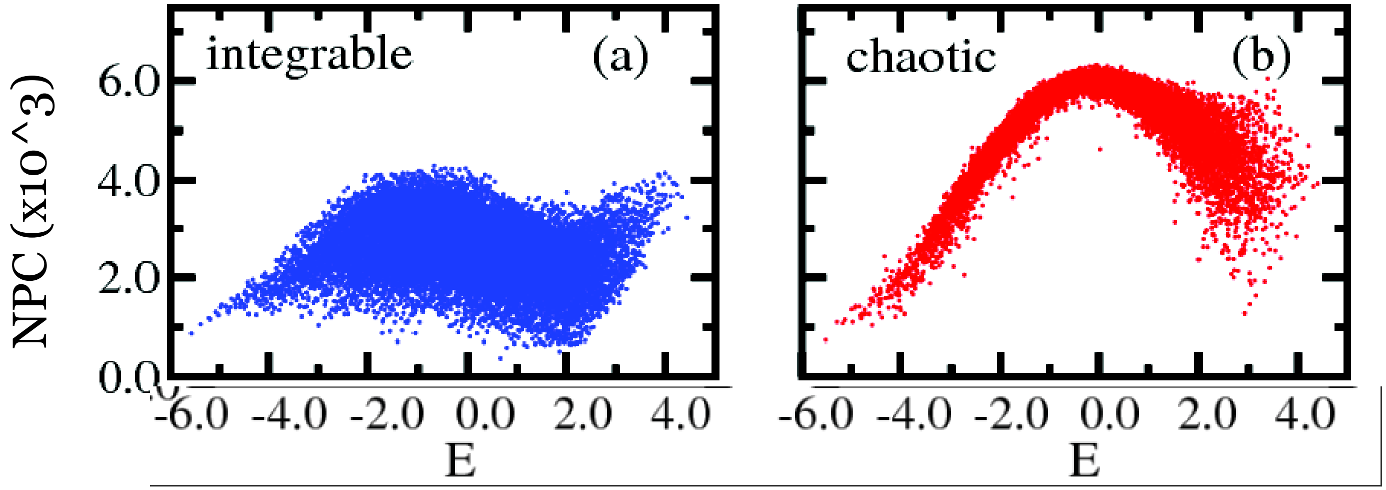
$$\psi_j = \sum_{k=1}^{\dim} a_{jk} \phi_k$$

$$NPC_j = \frac{1}{\sum_{k=1}^{\dim} | a_{jk} |^4}$$

Small NPC – localized state
Large NPC – delocalized state

Number of Principal Components

- Chaotic systems are significantly more delocalized
- Chaotic systems NPCs have smaller fluctuations



Symmetries

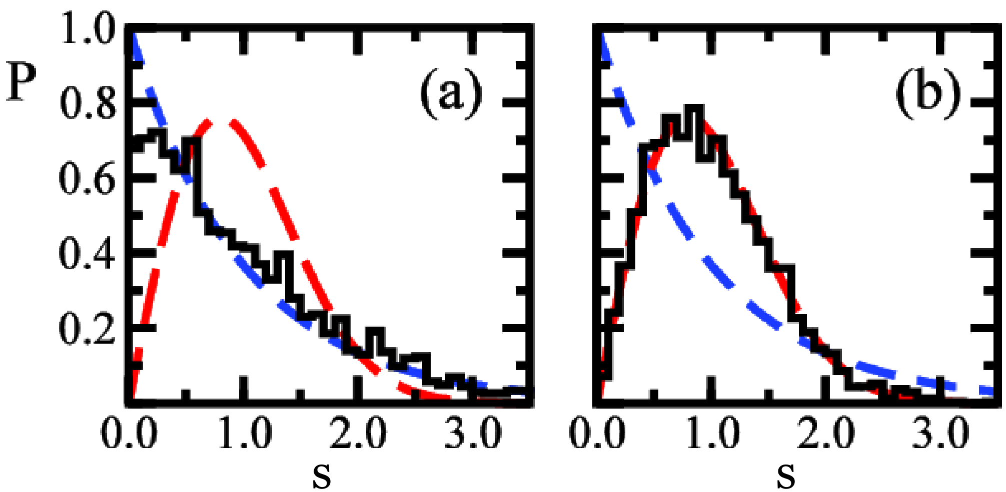
- We also study chaos in a system with no defect
- To drive it to chaos, we add next-nearest-neighbor couplings
- Parity
- Spin reversal
- Total spin

$$H = H_{NN} + \alpha H_{NNN}$$

$$H_{NN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]$$

$$H_{NNN} = \sum_{n=1}^{L-2} \left[J_z S_n^z S_{n+2}^z + J(S_n^x S_{n+2}^x + S_n^y S_{n+2}^y) \right]$$

Symmetries



A: L=14, 7 spins up
B: L=15, 5 spins up
Both: alpha = .5, J=Jz

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Thank you!