Controlling the Evolution of a Quantum System with Dynamical Decoupling Methods

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Outline

- Methods of quantum control to
  manipulate the dynamics of a system
- Dynamical Decoupling Methods
  - quantum information – store information, gates
  - quantum transport - obtain a desired dynamics
- Model: spin-1/2 chain
- Examples:
  - Disordered chaotic $\longrightarrow$ integrable
  - Frustrated chaotic $\longrightarrow$ integrable
  - Gapless $\longrightarrow$ gapped (vice-versa)

Spin-1/2 Heisenberg Model ⇓⇑⇓⇑

1D, L sites, open boundary conditions $\boxed{\hbar = 1}$

$$H = H_z + \beta_1 H_{NN} + \beta_2 H_{NNN}$$

$$S_n^{x,y,x} = \sigma_n^{x,y,z} / 2$$

Spin operators

On-site energy:

$$H_z = \sum_{n=1}^{L} \varepsilon_n S_n^z$$

Spin up: $+\varepsilon / 2$ ⇑

Spin down: $-\varepsilon / 2$ ⇓

Clean $\varepsilon_n = \varepsilon$

Defect: site with different Zeeman splitting (on-site disorder)
$$\begin{cases} \varepsilon_{n \neq m} = \varepsilon \\ \varepsilon_m = \varepsilon + d_m \end{cases}$$

Nearest-neighbor couplings

$$H = H_z + \beta_1 H_{NN} + \beta_2 H_{NNN}$$

$$\begin{cases} H_z = \sum_{n=1}^{L} \varepsilon_n S_n^z \\ H_{NN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right] \end{cases}$$

$$\Delta = \frac{J_z}{J}$$

Ising interaction

$$H_{zz} = \sum_{n=1}^{L-1} J_z S_n^z S_{n+1}^z$$

Flip-flop term

$$H_{XY} = \sum_{n=1}^{L-1} \left[J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]$$

Arrows……..

anisotropy

(we consider $\Delta > 0$)

- Disordered $\varepsilon_m \neq \varepsilon_n$ **CHAOTIC** ⟹ Clean $\varepsilon_n = \varepsilon$ **INTEGRABLE**
- Clean $\Delta < 1$ **GAPLESS** ⟹ Clean $\Delta > 1$ **GAPPED**

Next-Nearest-neighbor couplings

Clean frustrated system

$$H = \beta_1 H_{NN} + \beta_2 H_{NNN}$$

$$\boxed{\alpha = \frac{\beta_2}{\beta_1}}$$

$$\begin{cases} H_{NN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right] \\ H_{NNN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+2}^z + J(S_n^x S_{n+2}^x + S_n^y S_{n+2}^y) \right] \end{cases}$$

- NN+NNN $\alpha = 1$ **CHAOTIC** ⇒ NN $\alpha = 0$ **INTEGRABLE**
- $\alpha < \alpha_c = 0.24...$ **GAPLESS** ⇒ $\alpha < \alpha_c = 0.24...$ **GAPPED**

Dynamical Decoupling Methods

Inspired by techniques used in NMR spectroscopy

Sequences of pulses that rotate the spins and change the dynamics as desired

We consider pi-pulses – flip spins

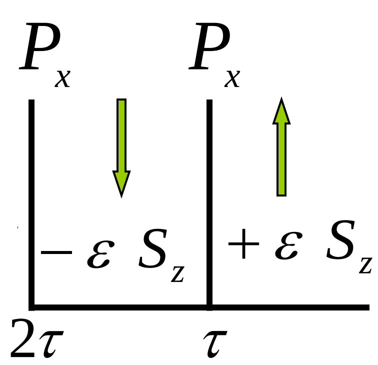
$$P_x = \exp[-i\pi\, S_x]$$

<u>Spin Echo:</u> avoid phase accumulation, freeze the evolution $U(t) \to \mathbb{1}$

$$H_0 = \varepsilon\; S_z$$

Free evolution: $\Psi(t) = U(t)\Psi(0) = e^{-i\varepsilon\, S_z t}\Psi(0)$

Under pulses:



$$P_x = \exp[-i\pi\, S_x]$$

$$U(2\tau) = P_x \exp[-i\varepsilon\; S_z\tau] P_x \exp[-i\varepsilon\; S_z\tau]$$

$$= (-1)\exp[i\varepsilon\; S_z\tau]\exp[-i\varepsilon\; S_z\tau] = -\mathbb{1}$$

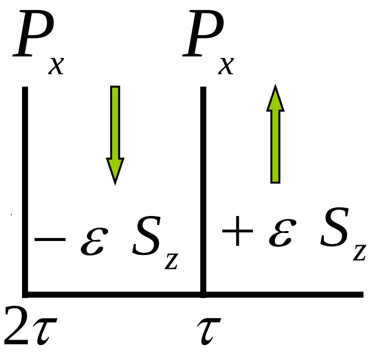
DD Method: Spin Echo

$U(t) \to \mathbb{1}$

$H_0 = \varepsilon \, S_z$

Free evolution: $\Psi(t) = U(t)\Psi(0) = e^{-i\varepsilon \, S_z t}\Psi(0)$

Under pulses:



$$P_x = \exp[-i\pi \, S_x]$$

$$U(2\tau) = P_x e^{i\varepsilon \, S_z \tau} P_x e^{-i\varepsilon \, S_z \tau}$$

$$= (-1)\exp[i\varepsilon \, S_z \tau]\exp[-i\varepsilon \, S_z \tau] = -\mathbb{1}$$

Suppressing on-site disorder

- Disordered **CHAOTIC** → Clean **INTEGRABLE**

$$H = H_z + H_{NN}$$

$$H = \varepsilon_{L/2} S^z_{L/2} + \sum_{n=1}^{L-1} \left[J_z S^z_n S^z_{n+1} + J (S^x_n S^x_{n+1} + S^y_n S^y_{n+1}) \right]$$

$$U(j\tau) = P_x e^{-i(+H_z + H_{NN})\tau} P_x e^{-i(+H_z + H_{NN})\tau} \ldots P_x e^{-i(+H_z + H_{NN})\tau} P_x e^{-i(+H_z + H_{NN})}$$

$$e^{-i(-H_z + H_{NN})\tau} e^{-i(+H_z + H_{NN})\tau} \ldots e^{-i(-H_z + H_{NN})\tau} e^{-i(+H_z + H_{NN})\tau}$$

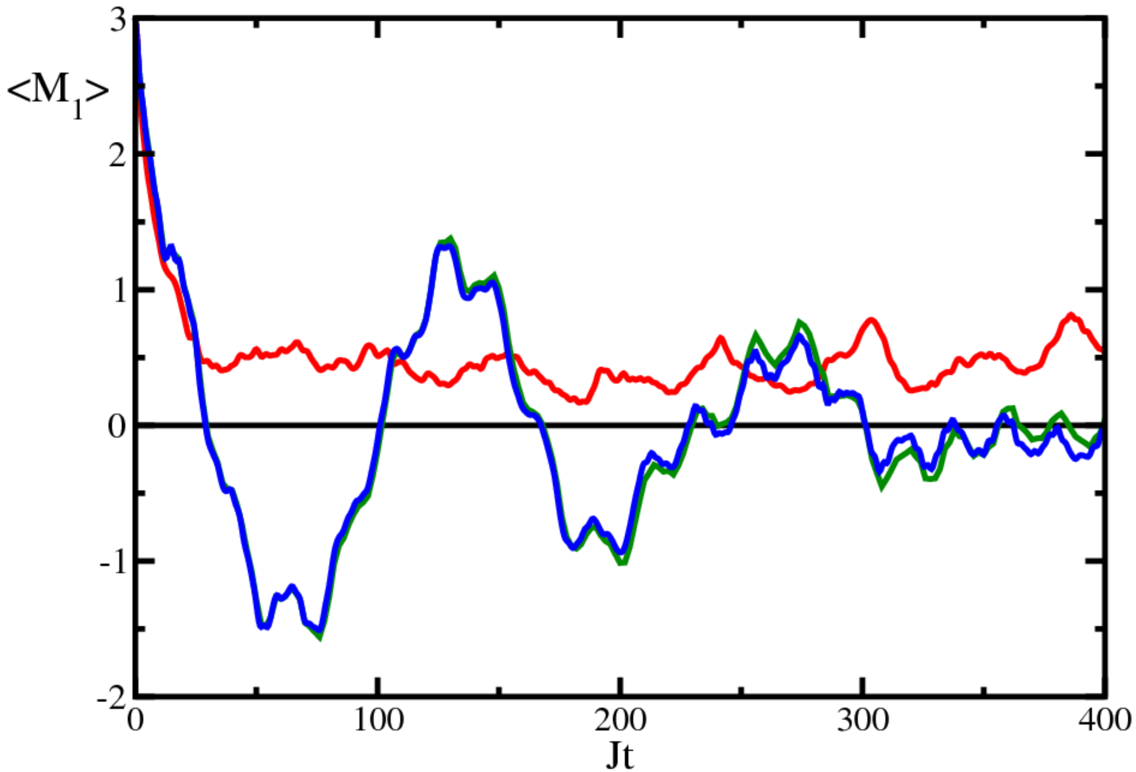
At $2j\tau$ the effects of on-site disorder disappear (to 1st order)

Suppressing on-site disorder

- Disordered **CHAOTIC** → Clean **INTEGRABLE**

$$< M(t) >=< \Psi(t) | \sum_{n=1}^{L/2} S_n^z |\Psi(t) >$$

$$| \Psi(0) >= | \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \ldots \downarrow\downarrow \rangle$$



Recover the transport behavior of the integrable system

$L = 12, \; up = down$

$\Delta = 1,$

$\varepsilon_7 / J = 0.35$

Dynamical Decoupling Methods

How to freeze the evolution? $U(t) \rightarrow \mathbb{1}$

$$H_{NN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]$$

Baker…

Average Hamiltonian theory

P_x^{odd}	P_x^{odd}	P_y^{odd}	P_x^{odd}
$H_4 =$	$H_3 =$	$H_2 =$	$H_1 =$
$- J\, S_n^x S_n^x$	$- J\, S_n^x S_n^x$	$+ J\, S_n^x S_n^x$	$+ J\, S_n^x S_n^x$
$+ J\, S_n^y S_n^y$	$- J\, S_n^y S_n^y$	$- J\, S_n^y S_n^y$	$+ J\, S_n^y S_n^y$
$- J_z S_n^z S_n^z$	$+ J_z S_n^z S_n^z$	$- J_z S_n^z S_n^z$	$+ J_z S_n^z S_n^z$
$T_c = 4\tau$	3τ	2τ	τ

$$U(T_c) = e^{-iH_4\tau} e^{-iH_3\tau} e^{-iH_2\tau} e^{-iH_1\tau} = e^{-i\overline{H}T_c}$$

$$= e^{-iT_c(\overline{H}^{(0)} + \overline{H}^{(1)} + \overline{H}^{(2)} + \ldots)}$$

$$\overline{H}^{(0)} = \frac{\tau(H_1 + H_2 + H_3 + H_4)}{T_c} = 0$$

$$\overline{H}^{(1)} \propto \tau^2, \;\; \overline{H}^{(2)} \propto \tau^3$$

Pulse sequence aims at achieving a desired DOMINANT term

Frustrated Chain $H_{NN} + H_{NNN} \rightarrow \frac{H_{NN}}{2}$

- NN+NNN $\alpha = 1$ **CHAOTIC** ➡ NN $\alpha = 0$ **INTEGRABLE**

Sequence of eight pulses to eliminate NNN couplings, but NN couplings remain.

$$P_1 = P_3 = \prod_{k=0}^{[(L-1)/4]} e^{-i\pi S^x_{1+4k}} \prod_{k=0}^{[(L-2)/4]} e^{-i\pi S^x_{2+4k}}$$

$$P_2 = P_4 = \prod_{k=0}^{[(L-3)/4]} e^{-i\pi S^y_{3+4k}} \prod_{k=0}^{[(L-4)/4]} e^{-i\pi S^y_{4+4k}}$$

$$P_5 = P_7 = \prod_{k=0}^{[(L-2)/4]} e^{-i\pi S^x_{2+4k}} \prod_{k=0}^{[(L-3)/4]} e^{-i\pi S^x_{3+4k}}$$

$$P_6 = P_8 = \prod_{k=0}^{[(L-1)/4]} e^{-i\pi S^y_{1+4k}} \prod_{k=0}^{[(L-4)/4]} e^{-i\pi S^y_{4+4k}}$$

NN	$0,\tau$	$\tau,2\tau$	$2\tau,3\tau$	$3\tau,4\tau$	$4\tau,5\tau$	$5\tau,6\tau$	$6\tau,7\tau$	$7\tau,8\tau$
$S^x_{2i+1}S^x_{2i+2}$	+	+	+	+	+	+	−	−
$S^x_{2i+1}S^x_{2i+2}$	+	+	−	−	+	+	+	+
$S^x_{2i+1}S^x_{2i+2}$	+	+	+	+	+	−	−	+
$S^x_{2i+1}S^x_{2i+2}$	+	−	−	+	+	+	+	+
$S^x_{2i+1}S^x_{2i+2}$	+	+	+	+	+	−	+	−
$S^x_{2i+1}S^x_{2i+2}$	[illegible]	−	+	−	+	+	+	+

NNN	$0,\tau$	$\tau,2\tau$	$2\tau,3\tau$	$3\tau,4\tau$	$4\tau,5\tau$	$5\tau,6\tau$	$6\tau,7\tau$	$7\tau,8\tau$
$S^x_{2i+1}S^x_{2i+2}$	+	+	−	−	+	+	−	−
$S^x_{2i+1}S^x_{2i+2}$	+	−	−	+	+	−	−	+
$S^x_{2i+1}S^x_{2i+2}$	+	−	+	−	+	−	+	−

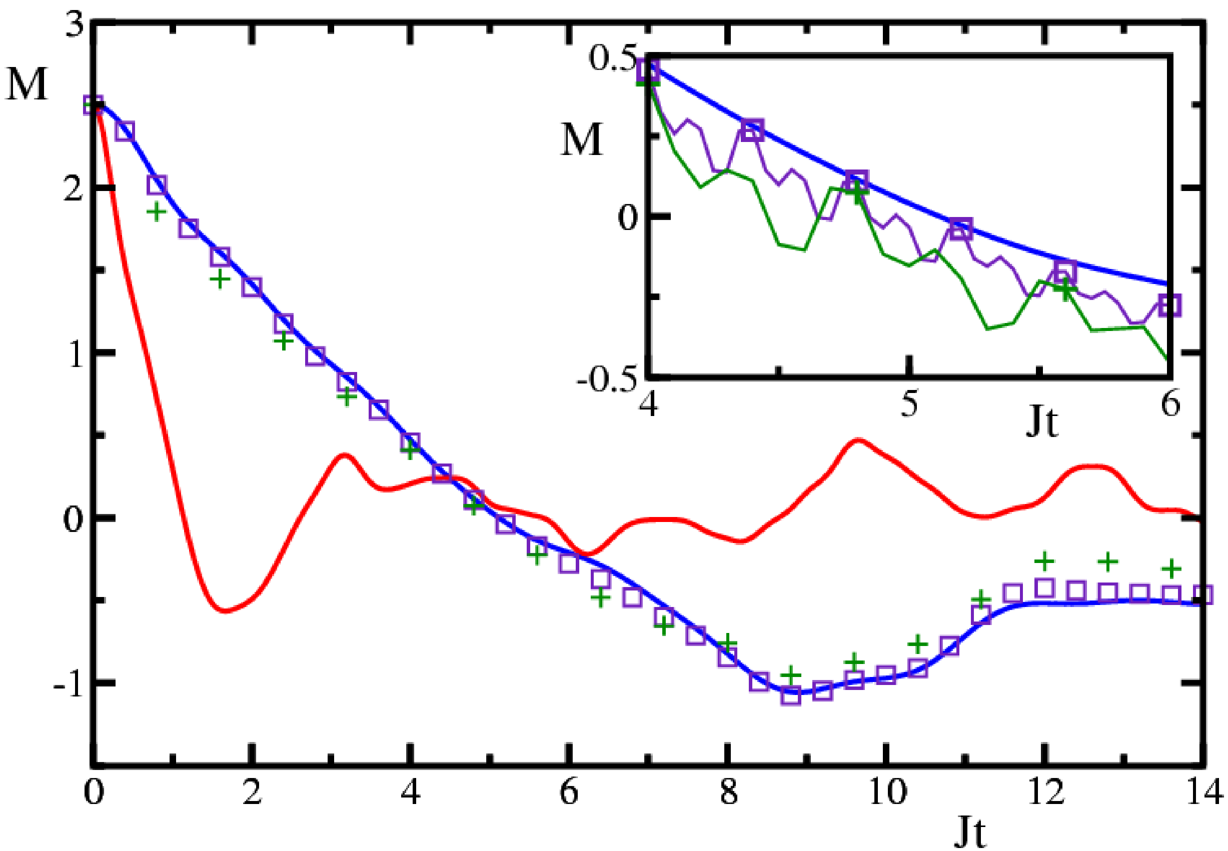
Frustrated Chain $H_{NN} + H_{NNN} \rightarrow \frac{H_{NN}}{2}$

NN+NNN $\alpha = 1$

□ **CHAOTIC**

NN $\alpha = 0$

INTEGRABLE



Recover the transport behavior of the integrable system, but slower

$L = 10, \ up = down$

$\Delta = 1$

$\tau = 0.05 J^{-1}$

$\tau = 0.1 J^{-1}$

Gapless to Gapped $\Delta < 1 \rightarrow \Delta > 1$

$$H_{NN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]$$

Vary the time interval between pulses to reduce effects of xx+yy

P_z^{odd} | P_z^{odd}

$H_2 =$
$- \Sigma J \, S_n^x S_n^x$
$- \Sigma J \, S_n^y S_n^y$
$+ \Sigma J_z S_n^z S_n^z$
(τ_2)

$H_1 =$
$+ \Sigma J \, S_n^x S_n^x$
$+ \Sigma J \, S_n^y S_n^y$
$+ \Sigma J_z S_n^z S_n^z$
(τ_1)

Original Hamiltonian with $\Delta = \dfrac{J_z}{J}$

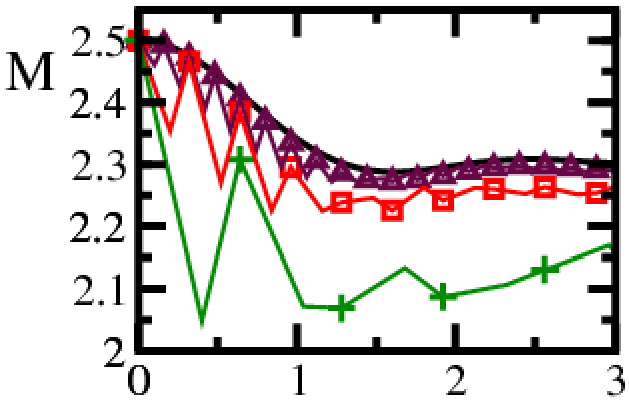
Under pulses, system evolves as $\Delta = \dfrac{J_z T_c}{J(\tau_1 - \tau_2)}$

$$T_c = \tau_1 + \tau_2$$

dominant

$$\bar{H}^{(0)} = \sum_{n=1}^{L-1} \left[\frac{J_{xy}(\tau_1 - \tau_2)}{T_c} (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) + J_z S_n^z S_{n+1}^z \right]$$

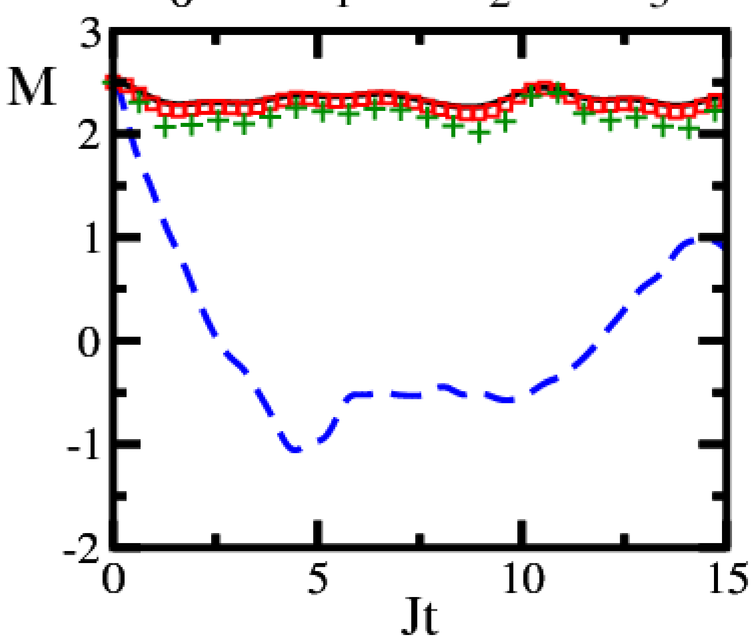
Gapless to Gapped $\Delta < 1 \rightarrow \Delta > 1$



$\tau_1 = 0.01J^{-1}, \tau_2 = 0.006J^{-1}$

$\tau_1 = 0.02J^{-1}, \tau_2 = 0.012J^{-1}$

$\tau_1 = 0.04J^{-1}, \tau_2 = 0.024J^{-1}$



$$\Delta = \frac{J_z T_c}{J(\tau_1 - \tau_2)} = 2$$ (insulator)

$\Delta = 1/2$ (metal)

$L = 10,\ up = down$

Conclusion

- Sequence of pulses to manipulate the dynamics of a quantum system.
  - Chaotic --- integrable
  - Gapless --- gapped
- Site addressability and variation of intervals between pulses.

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