Controlling the Evolution of a Quantum System with Dynamical Decoupling Methods

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Methods of quantum control to manipulate the dynamics of a system

Dynamical Decoupling Methods
- quantum information – store information, gates
- quantum transport - obtain a desired dynamics

Model: spin-1/2 chain

Examples:
- Disordered chaotic → integrable
- Frustrated chaotic → integrable
- Gapless → gapped (vice-versa)
Spin-1/2 Heisenberg Model

1D, L sites, open boundary conditions

\[ H = H_z + \beta_1 H_{NN} + \beta_2 H_{NNN} \]

On-site energy:

\[ H_z = \sum_{n=1}^{L} \epsilon_n S^z_n \]

\[ \hbar = 1 \]

\[ S^{x,y,z}_n = \frac{\sigma^{x,y,z}_n}{2} \]

Spin operators

Clean \( \epsilon_n = \epsilon \)

Defect: site with different Zeeman splitting (on-site disorder)

\[ \epsilon_{n \neq m} = \epsilon \\
\epsilon_m = \epsilon + d_m \]
Nearest-neighbor couplings

\[ H = H_z + \beta_1 H_{NN} + \beta_2 H_{NNN} \]

Ising interaction
\[ H_{zz} = \sum_{n=1}^{L-1} J_z S_n^z S_{n+1}^z \]
Flip-flop term
\[ H_{xy} = \sum_{n=1}^{L-1} [J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)] \]

\[ H_z = \sum_{n=1}^{L} \varepsilon_n S_n^z \]
\[ H_{NN} = \sum_{n=1}^{L-1} [J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)] \]
\[ \Delta = \frac{J_z}{J} \]

Arrows.......
Anisotropy
(we consider $\Delta > 0$)

Disordered \hspace{1cm} $\varepsilon_m \neq \varepsilon_n$

\larrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm}

Clean \hspace{2cm} $\varepsilon_n = \varepsilon$

\larrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm}

Clean \hspace{2cm} $\Delta > 1$

\larrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm}

Clean \hspace{2cm} $\Delta < 1$

\larrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm} \rightarrow \hspace{2cm}

CHAOTIC

GAPLESS
Next-Nearest-neighbor couplings

Clean frustrated system

\[ H = \beta_1 H_{NN} + \beta_2 H_{NNN} \]

\[ \alpha = \frac{\beta_2}{\beta_1} \]

- **NN+NNN** \( \alpha = 1 \)
  - **CHAOTIC**
- **\alpha < \alpha_c \approx 0.24...**
  - **GAPLESS**

\[ H_{NN} = \sum_{n=1}^{L-1} \left[ J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right] \]

\[ H_{NNN} = \sum_{n=1}^{L-1} \left[ J_z S_n^z S_{n+2}^z + J (S_n^x S_{n+2}^x + S_n^y S_{n+2}^y) \right] \]

- **NN** \( \alpha = 0 \)
  - **INTEGRABLE**
- **\alpha < \alpha_c \approx 0.24...**
  - **GAPPED**
Dynamical Decoupling Methods

Inspired by techniques used in NMR spectroscopy
Sequences of pulses that rotate the spins and change the dynamics as desired
We consider pi-pulses – flip spins

\[ P_x = \exp[-i\pi S_x] \]

**Spin Echo:** avoid phase accumulation, freeze the evolution \( U(t) \rightarrow 1 \)

\[ H_0 = \varepsilon S_z \]

Free evolution: \( \Psi(t) = U(t)\Psi(0) = e^{-i\varepsilon S_z t}\Psi(0) \)

Under pulses:

\[
\begin{array}{cc}
P_x & P_x \\
\downarrow & \uparrow \\
-\varepsilon S_z & +\varepsilon S_z \\
2\tau & \tau \\
\end{array}
\]

\[ U(2\tau) = P_x \exp[-i\varepsilon S_z \tau]P_x \exp[-i\varepsilon S_z \tau] \]
\[ = (-1) \exp[i\varepsilon S_z \tau] \exp[-i\varepsilon S_z \tau] = -1 \]
DD Method: Spin Echo

\[ H_0 = \varepsilon \, S_z \]

Free evolution:

\[ \Psi(t) = U(t)\Psi(0) = e^{-i\varepsilon S_z t} \Psi(0) \]

Under pulses:

\[ P_x = \exp[-i\pi S_x] \]

\[ U(2\tau) = P_x e^{i\varepsilon S_z \tau} P_x e^{-i\varepsilon S_z \tau} \]

\[ = (-1) \exp[i\varepsilon S_z \tau] \exp[-i\varepsilon S_z \tau] = -1 \]
Suppressing on-site disorder

Disordered **CHAOTIC**

\[ H = H_z + H_{NN} \]

\[ H = \varepsilon_{L/2} S_{L/2}^z + \sum_{n=1}^{L-1} [J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)] \]

\[ U(j\tau) = P_x e^{-i(H_z + H_{NN})\tau} P_x e^{-i(H_z + H_{NN})\tau} \ldots P_x e^{-i(H_z + H_{NN})\tau} P_x e^{-i(H_z + H_{NN})\tau} \]

\[ \ldots e^{-i(-H_z + H_{NN})\tau} e^{-i(+H_z + H_{NN})\tau} \ldots e^{-i(-H_z + H_{NN})\tau} e^{-i(+H_z + H_{NN})\tau} \]

At \( 2j\tau \) the effects of on-site disorder disappear (to 1\textsuperscript{st} order)
Suppressing on-site disorder

Disordered

\[ \text{CHAOTIC} \]

Clean

\[ \text{INTEGRABLE} \]

\[ < M(t) > = < \Psi(t) | \sum_{n=1}^{L/2} S^z_n | \Psi(t) > \]

\[ | \Psi(0) >= | \uparrow \uparrow \uparrow \uparrow \uparrow \downarrow \downarrow \downarrow \downarrow . \downarrow \downarrow \]

Recover the transport behavior of the integrable system

\[ L = 12, \ up = \ down \]

\[ \Delta = 1, \]

\[ \varepsilon_7 / J = 0.35 \]
Dynamical Decoupling Methods

How to freeze the evolution? \( U(t) \rightarrow 1 \)

\[
H_{NN} = \sum_{n=1}^{L-1} \left[ J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]
\]

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<thead>
<tr>
<th>( P_{x, odd} )</th>
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<tr>
<td>( H_4 = -J S_n^x S_n^x )</td>
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<td>( H_2 = +J S_n^x S_n^x )</td>
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\( T_c = 4\tau \quad 3\tau \quad 2\tau \quad \tau \)

Baker...

Average Hamiltonian theory

\[
U(T_c) = e^{-iH_4 \tau} e^{-iH_3 \tau} e^{-iH_2 \tau} e^{-iH_1 \tau} = e^{-i\overline{HT_c}}
\]

\[
\overline{H}^{(0)} = \frac{\tau(H_1 + H_2 + H_3 + H_4)}{T_c} = 0
\]

\[
\overline{H}^{(1)} \propto \tau^2, \quad \overline{H}^{(2)} \propto \tau^3
\]

Pulse sequence aims at achieving a desired DOMINANT term
Frustrated Chain \[ H_{NN} + H_{NNN} \rightarrow \frac{H_{NN}}{2} \]

\( \begin{array}{c}
\text{NN+NNN } \alpha = 1 \quad \text{CHAOTIC} \\
\text{NN } \alpha = 0 \quad \text{INTEGRABLE}
\end{array} \)

Sequence of eight pulses to eliminate NNN couplings, but NN couplings remain.

\[ P_1 = P_3 = \prod_{k=0}^{[(L-1)/4]} e^{-i\pi S_{i+4k}^x} \prod_{k=0}^{[(L-2)/4]} e^{-i\pi S_{2i+2}^x} \]

\[ P_2 = P_4 = \prod_{k=0}^{[(L-3)/4]} e^{-i\pi S_{3i+4k}^y} \prod_{k=0}^{[(L-4)/4]} e^{-i\pi S_{2i+2}^y} \]

\[ P_5 = P_7 = \prod_{k=0}^{[(L-2)/4]} e^{-i\pi S_{2i+4k}^x} \prod_{k=0}^{[(L-3)/4]} e^{-i\pi S_{2i+2}^x} \]

\[ P_6 = P_8 = \prod_{k=0}^{[(L-1)/4]} e^{-i\pi S_{i+4k}^y} \prod_{k=0}^{[(L-2)/4]} e^{-i\pi S_{2i+2}^y} \]
Frustrated Chain $H_{NN} + H_{NNN} \rightarrow \frac{H_{NN}}{2}$

$\alpha = 1$ \quad CHAOTIC

$\alpha = 0$ \quad INTEGRABLE

Recover the transport behavior of the integrable system, but slower

$L = 10$, $up = down$

$\Delta = 1$

$\tau = 0.05J^{-1}$

$\tau = 0.1J^{-1}$
Gapless to Gapped \( \Delta < 1 \rightarrow \Delta > 1 \)

\[
H_{NN}^{\Delta} = \sum_{n=1}^{L-1} \left[ J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right]
\]

Vary the time interval between pulses to reduce effects of \( xx + yy \)

\[
P_{z}^{\text{odd}}
\]

\[
\begin{align*}
H_2 &= -\Sigma J S_n^x S_n^x \\
-\Sigma J S_n^y S_n^y \\
+ \Sigma J z_n S_n^z S_n^z
\end{align*}
\]

\[
H_1 = \begin{align*}
& + \Sigma J S_n^x S_n^x \\
& + \Sigma J S_n^y S_n^y \\
& + \Sigma J z_n S_n^z S_n^z
\end{align*}
\]

Original Hamiltonian with \( \Delta = \frac{J_z}{J} \)

Under pulses, system evolves as \( \Delta = \frac{J_z T_c}{J (\tau_1 - \tau_2)} \)

\[
T_c = \tau_1 + \tau_2
\]

\[
\hat{H}^{(0)} = \sum_{n=1}^{\text{dominant}} \left[ \frac{J_{xy} (\tau_1 - \tau_2)}{T_c} (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) + J_z S_n^z S_{n+1}^z \right]
\]
Gapless to Gapped $\Delta < 1 \rightarrow \Delta > 1$

![Graph showing M vs. Jt]

$\tau_1 = 0.01 J^{-1}, \tau_2 = 0.006 J^{-1}$

$\tau_1 = 0.02 J^{-1}, \tau_2 = 0.012 J^{-1}$

$\tau_1 = 0.04 J^{-1}, \tau_2 = 0.024 J^{-1}$

$\Delta = \frac{J T_c}{J (\tau_1 - \tau_2)} = 2$ (insulator)

$\Delta = 1/2$ (metal)

$L = 10, \ up = down$
Conclusion

- Sequence of pulses to manipulate the dynamics of a quantum system.
  - Chaotic --- integrable
  - Gapless --- gapped

- Site addressability and variation of intervals between pulses.

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