Interplay between interaction and (un)correlated disorder in one-dimensional many-particle systems: delocalization and global entanglement

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Introduction

- Interplay between disorder and interaction
- Uncorrelated and long range correlated disorder
- Chaos

Outline

- Model
- Effect on half filled chain
- Comparison with dilute chain

Heisenberg Spin-1/2 Model

One dimensional Hamiltonian $$H = H_0 + H_{\text{int}} + H_{XY}$$

On Site Energy $$H_0 = \tfrac{1}{2}\sum_{n=1}^{L}\Omega_n \sigma_n^z$$

$\sigma_n^{x,y,z}$ Pauli matrices

$\Omega_n = \omega + \omega_n$ Zeeman splitting

$\omega_n = d\varepsilon_n$ d=0: Clean System

$$\varepsilon_n = \sum_{k=1}^{L-1}\left[\sqrt{k^{-\alpha}\left|\frac{2\pi}{L}\right|^{1-\alpha}}\cos\left(\frac{2\pi nk}{L} + \phi_k\right)\right]$$

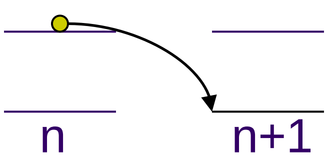
$\alpha = 0$ Uncorrelated disorder

$\alpha > 0$ Correlated disorder

$\phi \in [0, 2\pi]$ Uniform random numbers

Interaction Term $$H_{\text{int}} = \sum_{n=1}^{L-1}\frac{J\Delta}{4}\sigma_n^z \sigma_{n+1}^z$$

Hopping Term $$H_{XY} = \sum_{n=1}^{L-1}\tfrac{J}{4}\left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right)$$



Delocalization & Entanglement

Spatial Delocalization

$$\left|\Psi_j\right\rangle = \sum_{k=1}^{N} c_j^k \left|\phi^k\right\rangle \Rightarrow NPC_j \equiv \frac{1}{\sum_{k=1}^{N}\left|c_j^k\right|^4}$$

$\boxed{\text{Large NPC: delocalization}}$

Basis: eigenstates of $H_0 = \frac{1}{2}\sum_{n=1}^{L}\Omega_n \sigma_n^z$

Chaoticity

$$\eta \equiv \frac{\int_0^{s_0}\left[P(s) - P_{WD}(s)\right]ds}{\int_0^{s_0}\left[P_P(s) - P_{WD}(s)\right]ds}$$

Wigner-Dyson distribution

$$P_{WD}(s) = \pi s/2 \exp\left(-\pi s^2/4\right)$$

$\eta \to 0$ Chaos

Poisson distribution

$$P_P(s) = \exp(-s)$$

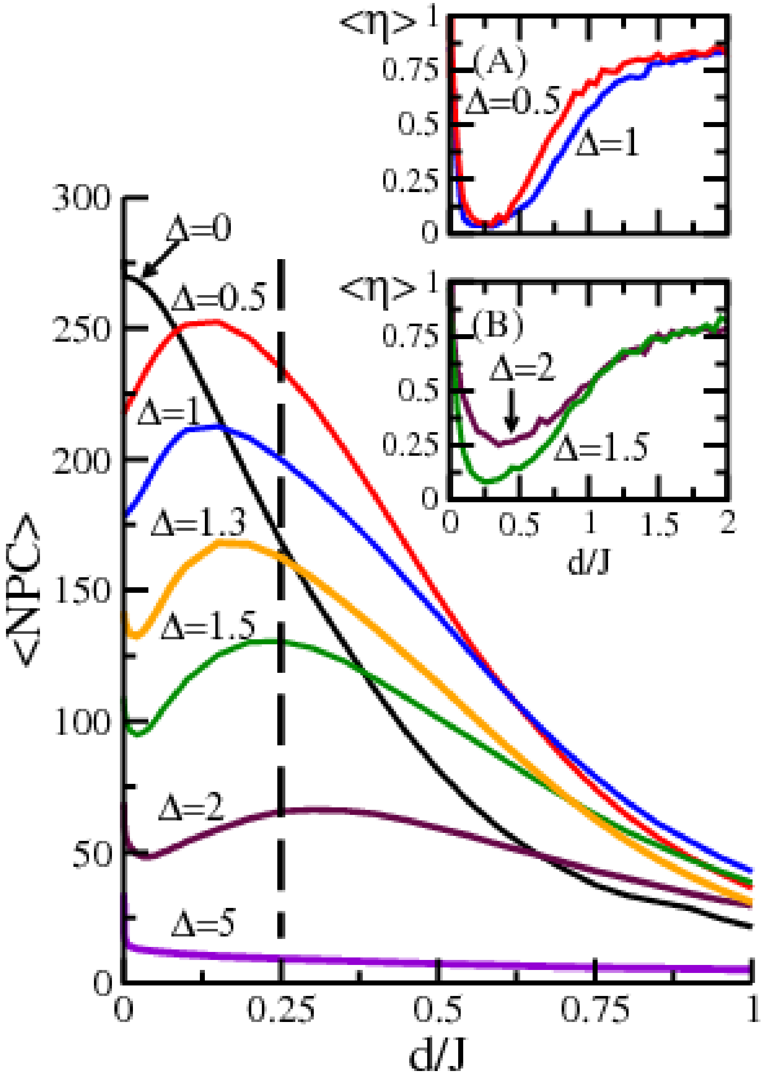
$\eta \to 1$ Integrable

Quantum Correlation - Global Entanglement

$$Q_j = 2 - \frac{2}{L}\sum_{n=1}^{L} Tr(\rho_n^2) \Rightarrow Q_j = 1 - \sum_{n=1}^{L}\left|\left\langle \Psi_j \left| \sigma_n^z \right| \Psi_j \right\rangle\right|^2$$

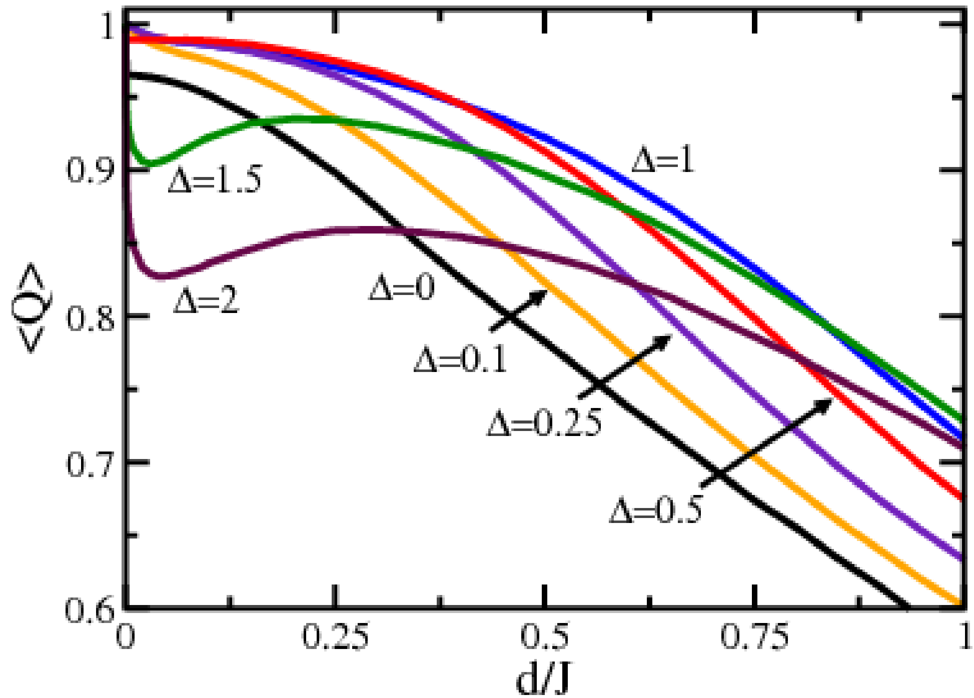
$\boxed{\text{Q=1: maximum global entanglement}}$

Half Filled Chains: Uncorrelated Disorder



Average over 20 realizations

L=12 sites, M=6 excitations

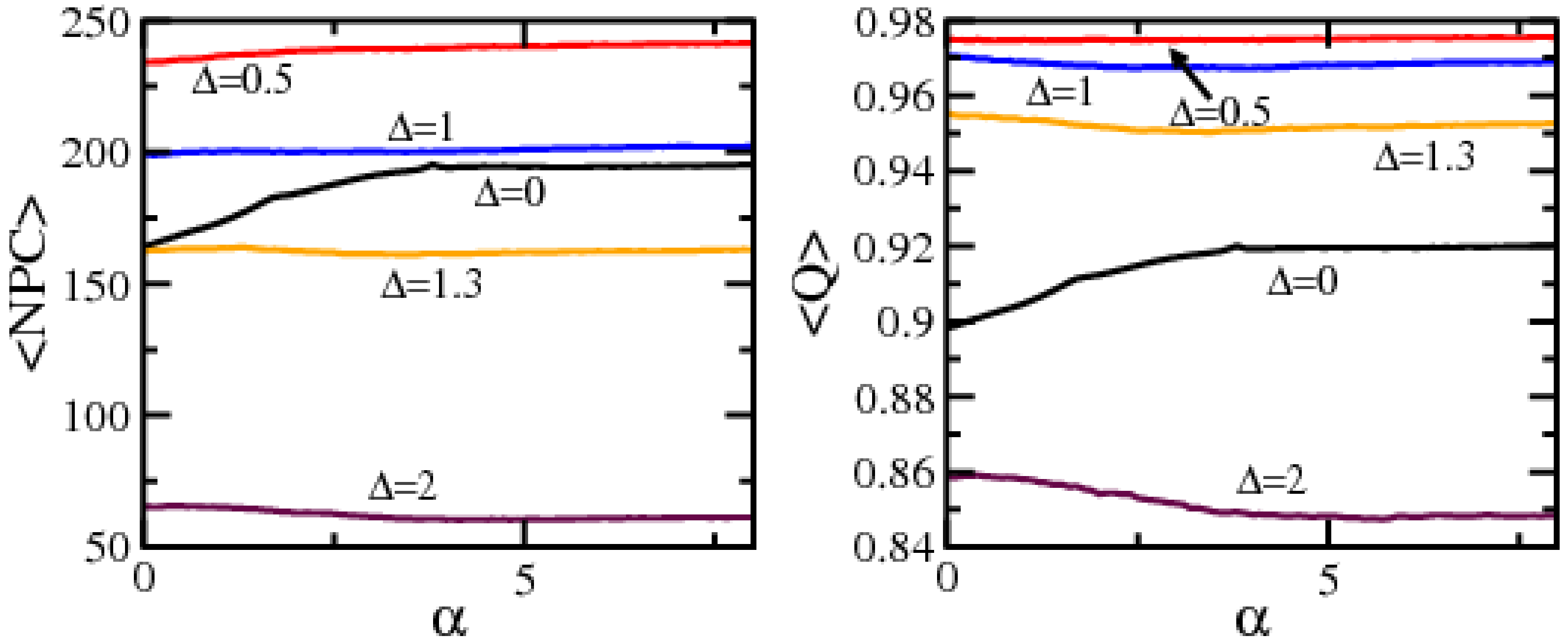


- In the gapless phase $\Delta < 1$ NPC peaks in the chaotic region
- Interaction is essential for entanglement

Long Range Correlated Disorder

$d/J = \frac{1}{4}$ ⇒ Chaotic Region $\alpha \geq 0$

L=12 sites, M=6 excitations



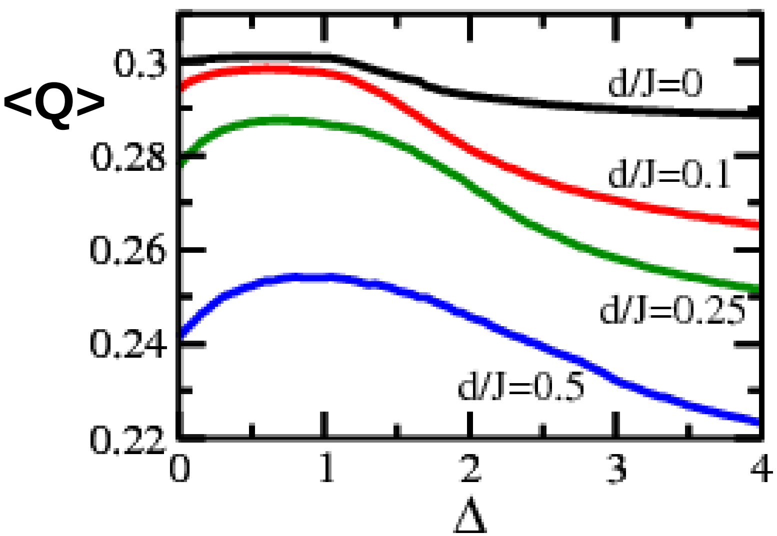
$\Delta < 1$: Delocalization and entanglement **increase** with correlated disorder

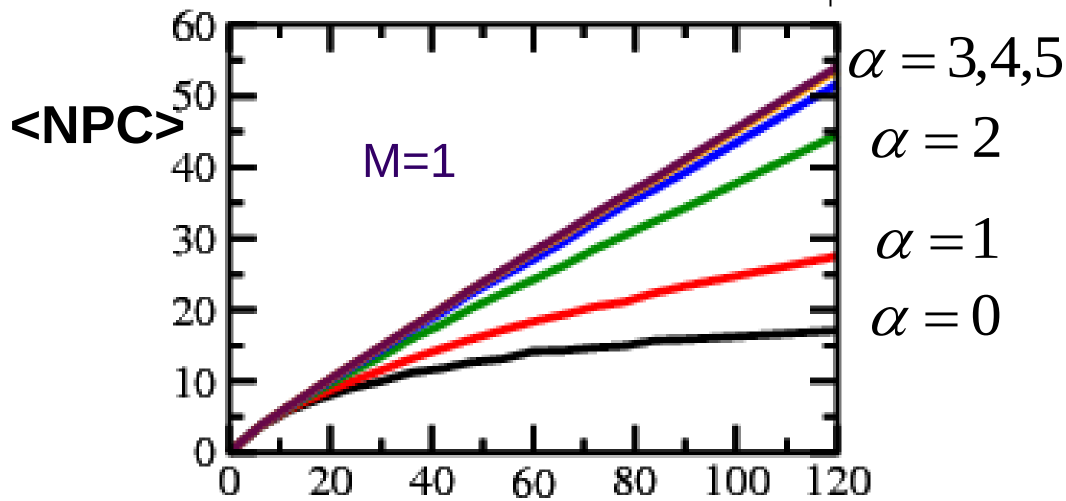
$\Delta > 1$: Delocalization and entanglement **decrease** with correlated disorder

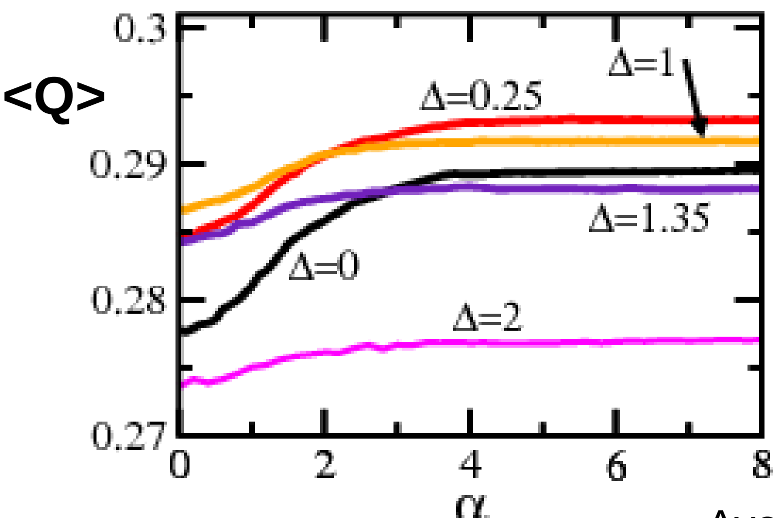
Average over 20 realizations

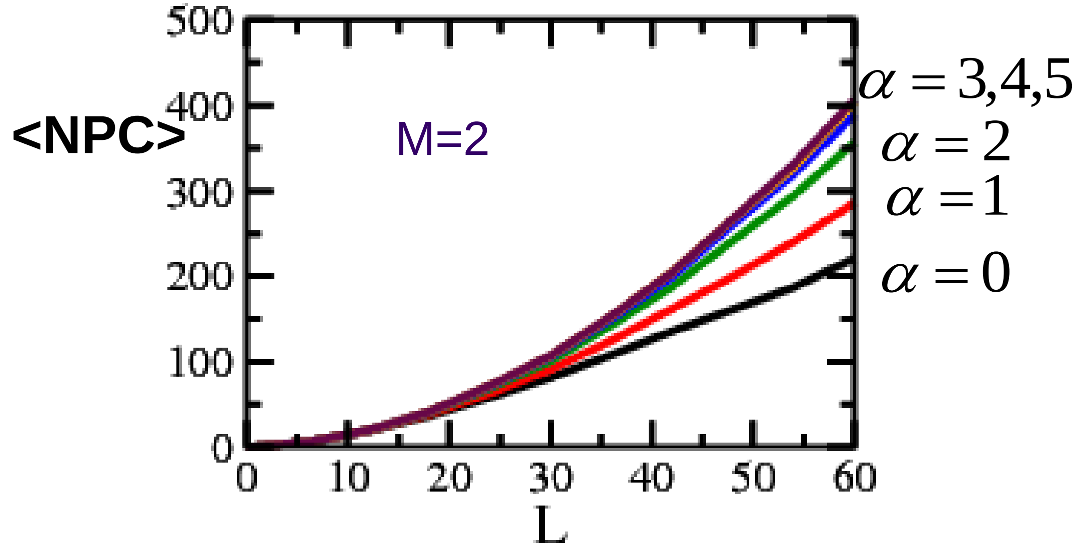
Dilute Limit

L=24, M=2 d/J=1/4 $\Delta=1$









Average over 20 realizations

Conclusions

Half Filled Chains

- Disorder + Interaction $\longrightarrow$ Chaos

  More Delocalization
- Entanglement more affected by interaction than chaos
- Correlated disorder in chaotic region
  - $\Delta \leq 1$ - Increase of NPC, Q
  - $\Delta > 1$ - Decrease of NPC, Q

Dilute Chains

- Correlated disorder is more efficient in increasing NPC, Q
- No sign of Anderson localization for finite chains with M=2

Future

- Transport behavior
- To include long-range interactions
- Experimental verification

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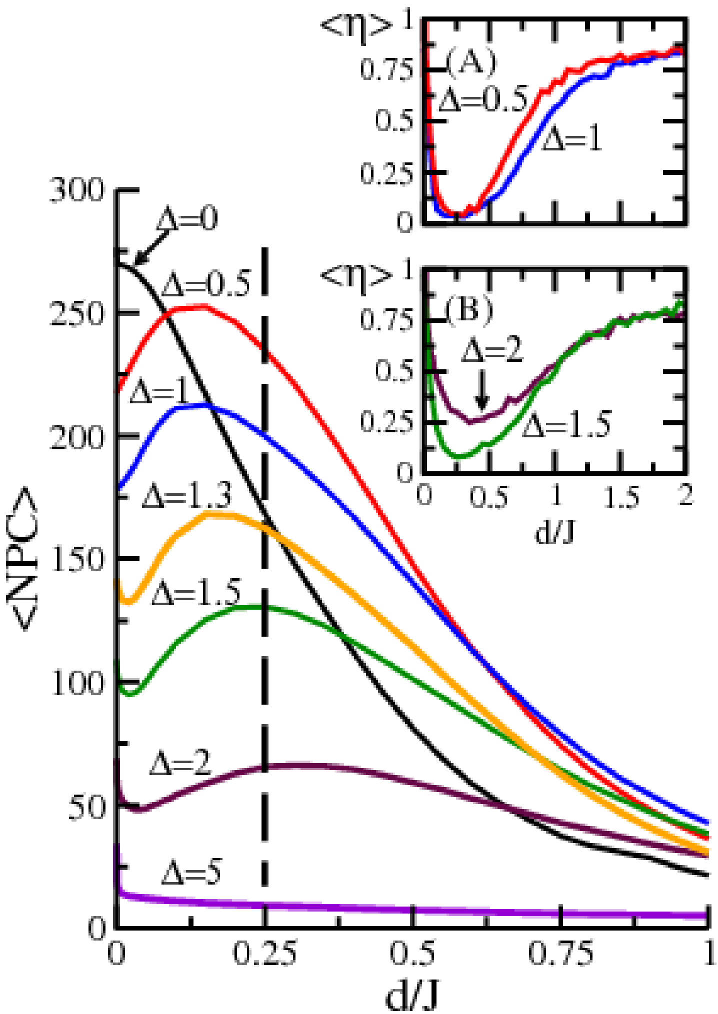
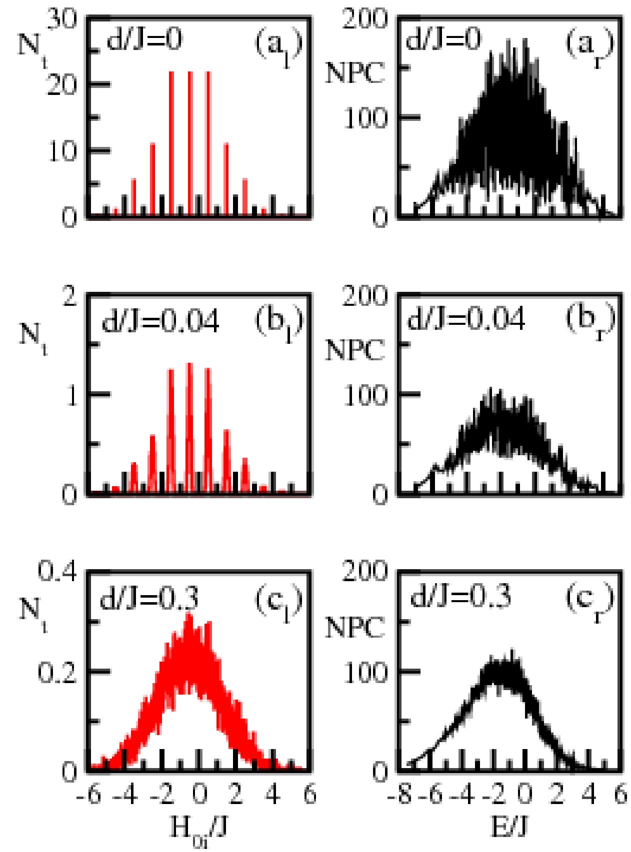
How histograms help understanding the results



Nt: number of states with a given diagonal energy

$\Delta = 2$

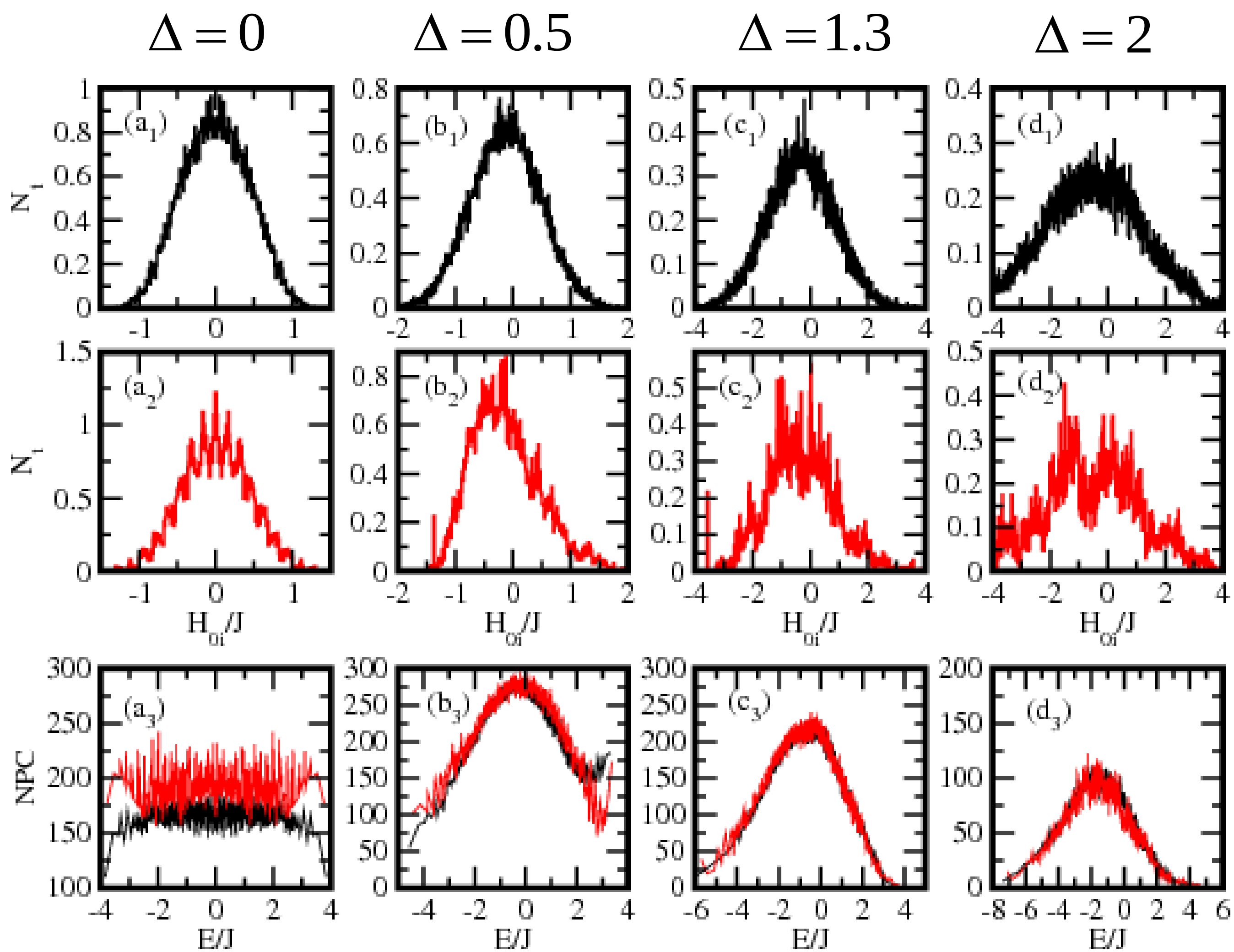


How histograms help understanding the results

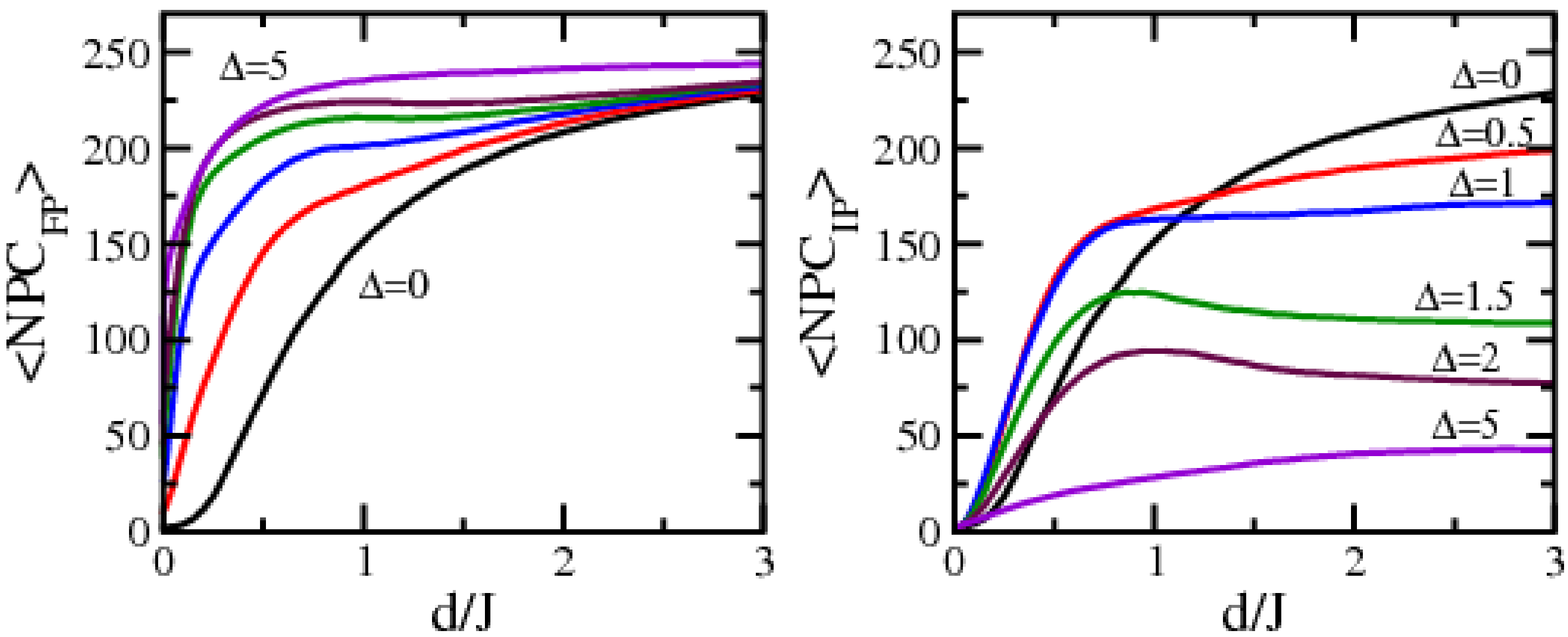
Black: uncorrelated disorder

Red: correlated disorder $\alpha = 10$

Nt: number of states with a given diagonal energy



NPC computed in different basis



FP: basis consisting of states of the clean Hamiltonian with no Ising interaction
IP: basis consisting of states of the clean Hamiltonian