Quantum Chaos: An introduction via chains of interacting spins 1/2

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What is quantum chaos?

- Classical chaos
  - Hypersensitivity to initial conditions
  - Dynamical billiard: billiard table with no friction and elastic collisions.
  - Depending on the shape: chaos
  - In phase space, the trajectories of two particles with very close initial conditions will diverge exponentially in time (rate=Lyapunov exponent)
What is quantum chaos?

• Classical chaos
  ▫ Hypersensitivity to initial conditions

• Quantum chaos
  ▫ Cannot use hypersensitivity due to Heisenberg’s Uncertainty Principle
  ▫ Classical systems are a limit of quantum systems
  ▫ Quantum billiards: distribution of neighboring energy levels depends on the billiard’s classical counterpart.
Level spacing distribution

Histogram of the spacings between neighboring energy levels

Energy levels

\[ E_0 \quad E_1 \quad E_2 \quad E_3 \quad E_4 \]

Energy spacings

\[ s_1 \quad s_2 \quad s_3 \quad s_4 \]
Level spacing distribution

• When classical billiard was chaotic, the energy levels of the quantum billiard are highly correlated and repel each other.

• The distribution is given by the Wigner-Dyson distribution, \[ P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4}. \]

• In an integrable (non-chaotic) system, the energy levels may cross \[ P(s) = e^{-s}. \]

• The distribution is Poissonian,
Level spacing distribution

- When classical billiard was chaotic, the energy levels of the quantum billiard are highly correlated and repel each other.
- The distribution is given by the Wigner-Dyson distribution,
  \[ P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4} \]
The system

- We study a 1D system of spins-1/2 with L sites
- Each site contains either a spin up or a spin down
- We use a chain with L/3 spins up (excitations)
The Hamiltonian

- Our system is described by the Hamiltonian

$$H = H_z + H_{NN}$$

$$H_z = \sum_{n=1}^{L} \varepsilon_n S_n^z$$

Clean \hspace{1cm} \varepsilon_n = \varepsilon

Defect -- site with different Zeeman splitting:

$$\begin{cases} 
\varepsilon_{n\neq m} = \varepsilon \\
\varepsilon_m = \varepsilon + d_m 
\end{cases}$$

$$H_{NN} = \sum_{n=1}^{L-1} \left[ J_z S_n^z S_{n+1}^z + J \left( S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) \right]$$

$$H_{zz} = \sum_{n=1}^{L-1} J_z S_n^z S_{n+1}^z$$

Ising Interaction

$$H_{XY} = \sum_{n=1}^{L-1} \left[ J \left( S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) \right]$$

Flip-Flop Term
Level spacing distribution

Histogram of the spacings between neighboring energy levels

<table>
<thead>
<tr>
<th>Energy levels</th>
<th>Energy spacings</th>
</tr>
</thead>
<tbody>
<tr>
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Level spacing distribution

- In chaotic systems, the energy levels are highly correlated and repel each other.
- The distribution is given by the Wigner-Dyson distribution, $P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4}$.
- In an integrable (non-chaotic) system, the energy levels may cross.
- The distribution is Poissonian, $P(s) = e^{-s}$. 
Level spacing distribution

- When the defect is placed on site 1, we obtain the Poissonian distribution, corresponding to an integrable system.
- When the defect is placed on site $L/2$, we obtain the Wigner-Dyson distribution, corresponding to a chaotic system.

$L = 15$
5 spins up
$J_z = .5 J$
$\epsilon_1, \epsilon_{L/2} = .5 J$
Number of Principal Components

• NPC is a measure of the delocalization of eigenstates
  ▫ It gives the number of basis vectors $\Phi$ which contribute to each eigenstate

$$\psi_j = \sum_{k=1}^{\text{dim}} a_{jk} \phi_k$$

$$\text{NPC}_j = \frac{1}{\sum_{k=1}^{\text{dim}} |a_{jk}|^4}$$

Small NPC – localized state
Large NPC – delocalized state
Number of Principal Components

- Chaotic systems are significantly more delocalized
- Chaotic systems NPCs have smaller fluctuations
Symmetries

- We also study chaos in a system with no defect
- To drive it to chaos, we add next-nearest-neighbor couplings
- Parity
- Spin reversal
- Total spin

\[ H = H_{NN} + \alpha H_{NNN} \]

\[ H_{NN} = \sum_{n=1}^{L-1} \left[ J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right] \]

\[ H_{NNN} = \sum_{n=1}^{L-2} \left[ J_z S_n^z S_{n+2}^z + J (S_n^x S_{n+2}^x + S_n^y S_{n+2}^y) \right] \]
Symmetries

A: $L=14$, 7 spins up
B: $L=15$, 5 spins up
Both: $\alpha = .5$, $J=J_z$
Acknowledgements

- Henry Kressel Research Scholarship, for funding this project

Thank you!