Numerical Explorations in the Non-Linear Schrodinger Equation with Non-Symmetrical Gaussian Initial conditions

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1 Introduction

The Nonlinear Schrodinger (NLS) equation

$$-i\frac{\partial\psi}{\partial t}+\frac{1}{2}(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2})+f(\|\psi\|^2)\psi=0$$

is a nonlinear partial differential equation (PDE) with physical applications in optics, fluid flow, and other fields . The nonlinear component of the equation can appear in many forms. When $f(\|\psi\|^2)=\|\psi\|^2$ the beam will self diffract if the power falls below the critical power, and will self-focus until collapse if above the critical power. This case is termed "Kerr non-linearity". Instead, we focused on saturated non-linearity, with $f(\|\psi\|^2)=\frac{\|\psi\|^2}{1+\gamma\|\psi\|^2}$. In particular, we focused on the case of two transverse dimensions (x,y; spatial dimensions) against the dimension of propagation (expressed in the equation as t; the time dimension, this is the third spatial dimension z). Saturated non-linearity acts as a limitation on the nonlinear component of the equation to prevent it from blowing up as it $\|\psi\|$ grows.

We began our research by following the method laid out by Gatz and Herrmann[1] to solve for the spatial solution to the NLS. We applied the iterative method described in the paper to solve for the initial conditions (IC) of this PDE given our chosen parameters (γ) and constraints. We replicated figures 1-3 from the paper to ensure our replication was sound. Our methodology was to run through a range of possible ρ_0 for each γ and to observe and pick the values which fulfil the constraint $\rho_1=\frac{1}{2}\rho_0$. Most γ values had two corresponding ρ_0 which fulfilled the constraint. We used these ρ_0 values to find ρ vectors,

[1] Propagation of optical beams and the properties of two-dimensional spatial solitons in media with a local saturable nonlinear refractive index, Optical Society of America B, Vol. 14, No. 7/July 1997/J. pg. 1795-1806

which serve as the initial condition for this equation. Such an IC physically corresponds to the the initial state of the beam, which we use as a basis to step forward in time and approximate the development of the beam.

This leads to the second part of our research. We plugged our IC into a split-step method to step forward in time and approximate the stationary solution $\psi = e^{i\beta t} f(x, y)$ to our PDE for any given time. We obtained the power of the beam from our soliton and constructed a Gaussian function of two dimensions whose power is equivalent to the power of the solution. We ran the split-step method on the Gaussian approximation, plugging in different values for variables which affect the symmetry of the beam as well as the relative widths of the Gaussians. In all, we obtained data for 96 unique beams. While running these methods, we collected data about how the amplitude and widths, in both dimensions, changed in any one beam over time. We observed that there are two internal modes relating to the widths of this beam. In order to further analyse this data, we automated a process to take Fourier transforms of the amplitude and widths, and collect information concerning their frequencies. We summarize and present these data pertaining to the frequencies in tables.

Further research would involve running the codes with finer steps — we used 512 points on our intervals, but are interested in running the codes for 1024 and 2048 points. Further analysis must be done on the patterns that occur in the amplitude and widths as they change in the beam over time.