Numerical Explorations in the Non-Linear Schrodinger Equation

Yedidya Moise (Yeshiva University), Yonah Moise (Yeshiva University)
Prof. Jeremy Schiff, Department of Mathematics, Bar-Ilan University
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ABSTRACT

The Nonlinear Schrodinger equation is a partial differential equation (PDE) whose principle application is to the propagation of a beam of light. Saturated non-linearity acts as a limitation on the nonlinear component of the equation to prevent it from blowing up. After reproducing initial conditions from Gatz and Herrmann (which are based on certain constraints), the split-step method was applied to step forward in time and approximate the solution to this PDE for any given time. We constructed a Gaussian function of two dimensions (with a power equivalent to the power of the solution), and ran the split-step method on this function to study it as an approximation of the solution.

INTRODUCTION

The Nonlinear Schrodinger (NLS) equation

$$-i\frac{\partial\psi}{\partial t}+\frac{1}{2}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+f(\|\psi\|^2)\psi=0$$

is a nonlinear partial differential equation (PDE) with physical applications in optics, fluid flow, and other fields.

- We focused on saturated non-linearity, which prevents the nonlinear component of the equation from blowing up as $||\psi||$ grows. $f(||\psi||^2)=\frac{||\psi||^2}{1+\gamma||\psi||^2}$.
- We focused on the case of spatial dimensions (x, y) against the dimension of propagation (time dimension, 3rd spatial dimension).

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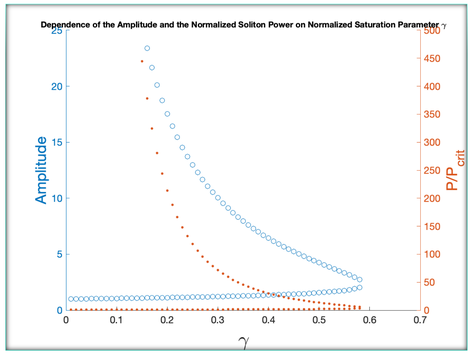
REFERENCES

[1] YU/BIU Summer 2022 program abstract book, https://www.biu.ac.il/sites/default/files/inline-files/BIU-YU%20Journal%202022.pdf.
[2] Gatz, S. and Herrmann, J. July 1997. “Propagation of optical beams and the properties of two-dimensional spatial solitons in media with a local saturable nonlinear refractive index,” Optical Society of America B, Vol. 14, No. 7. pp. 1795-1806.

METHODOLOGY

Stage 1:

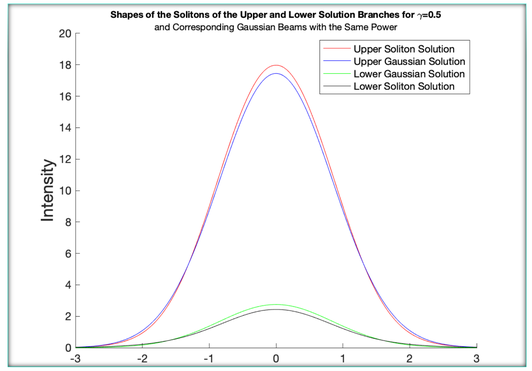
- We applied the iterative method described in the paper by Gatz and Herrmann to solve for the initial conditions (IC).
- We ran through a range of possible ρ_0 for each γ parameter and picked the values which fulfil the constraint $\rho_1 = \frac{1}{2}\rho_0$. Most γ values had two corresponding ρ_0.
- We used these ρ_0 values to find ρ vectors, which serve as the IC for this equation. This IC physically corresponds to the the initial state of the beam. [2]



METHODOLOGY

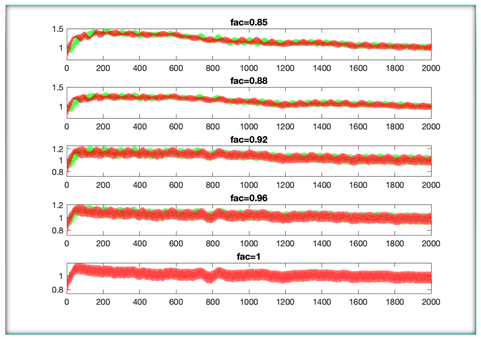
Stage 2:

- We plugged our IC (ρ) into a split-step method to step forward in time and approximate the development of the beam with the stationary solution $\Psi = e^{i\beta t} f(x, y)$.
- We constructed a Gaussian function of two dimensions whose power is equivalent to the power of the soliton solution.
- We ran the split-step method on the Gaussian approximation, plugging in different values for variables which affect the symmetry of the beam as well as the relative widths of the Gaussians.



RESULTS

- We obtained data for 96 unique beams.
- The data concerned how the amplitude and widths, in both dimensions, changed in any one beam over time.
- We observed that there are two internal modes relating to the widths of this beam.



- We took Fourier transforms of the amplitude and widths, and collected information concerning their frequencies. These data are awaiting analysis.

CONCLUSIONS & FUTURE WORK

- Further research would involve running the codes with finer steps — we used 512 points on our intervals, but are interested in running the codes for 1024 and 2048 points.
- Further analysis must be done on the patterns that occur in the amplitude and widths as they change in the beam over time.