Investigating New Resonance Transitions in Oterma's Orbit to Explain The Latest Observations

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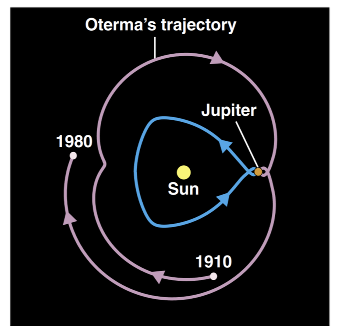
Introduction

- 39P/Oterma is a Jupiter-family comet whose heliocentric orbit changes substantially after close interactions with Jupiter.
- Three-Body-Problem involves three objects (Sun-Jupiter-Oterma here) under the gravitational influence of one another.
- Classical work in the planar circular restricted three-body problem (PCRTBP), considering circular orbit of Jupiter, explains the familiar transition between the 3:2 (3 revolutions of Oterma while 2 revolutions of Jupiter around the sun) resonance inside Jupiter's orbit and the 2:3 resonance outside it through heteroclinic connections between the Lyapunov families near the collinear points L1 and L2.
- The latest observations (2011) suggest that in addition to this pattern, there are episodes of close encounter with Saturn.
- We study the planar elliptic restricted three-body problem (PERTBP) as a small time-periodic perturbation of the PCRTBP and ask whether it opens new resonance pathways.

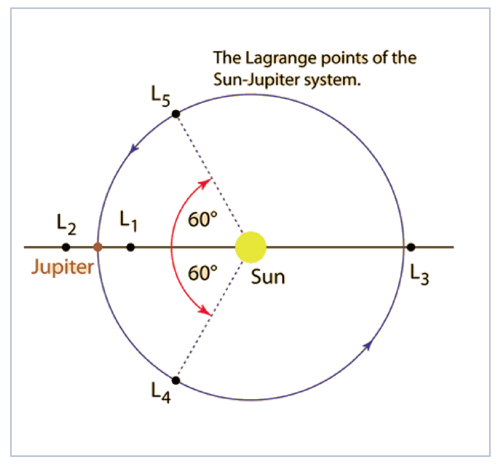


[1]

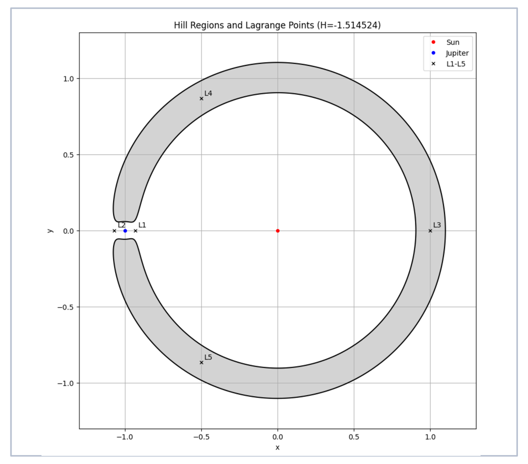
Aim: Identify new resonances, track the transfer geometry across a relevant energy window, and test whether small elliptic forcing can accumulate into the energy drift needed for Oterma's later transitions.

Strategy

- In the rotating Sun-Jupiter frame, Oterma is modeled as a massless body. For energies of Oterma (kinetic + potential) =h in [h1, h2] , with h1 = -1.514523604771845 (energy computed from observed data) and h2 = -1.513, we compute Lyapunov periodic orbits (tiny closed paths born from an equilibrium point due to oscillation) near L1 and L2, then intersect their stable and unstable manifolds with the Poincare sections y = 0 and x = mu - 1.
- Symmetric homoclinic points are converted to Delaunay coordinates so that the instantaneous semimajor axis a and period T = a^(3/2) reveal the accessible resonances. Repeating the computation at h1 and h2 lets us compare the resonance structure near the two neck regions and verify the transfer geometry from the inner region to the outer region.
- We then perturb to the PERTBP and use the scattering map to estimate the O(epsilon) energy change produced by repeated homoclinic excursions.



PCRTBP Setup for Sun-Jupiter-Oterma. L1, L2, L3, L4, and L5 are Equilibrium points.



Hill Region for H(L2)<h<H(L3). The grey region is the forbidden region. Oterma can move in the white region.

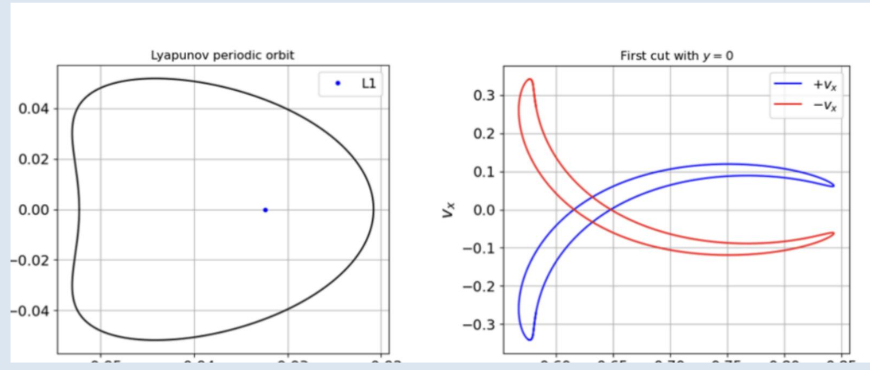
$$H_0(x,y,p_x,p_y) = \frac{(p_x+y)^2+(p_y-x)^2}{2} - \Omega(x,y)$$

$$H_\varepsilon(x,y,p_x,p_y,t) = \frac{(p_x+y)^2+(p_y-x)^2}{2} - \frac{\Omega(x,y)}{1+\varepsilon\cos(t)}$$

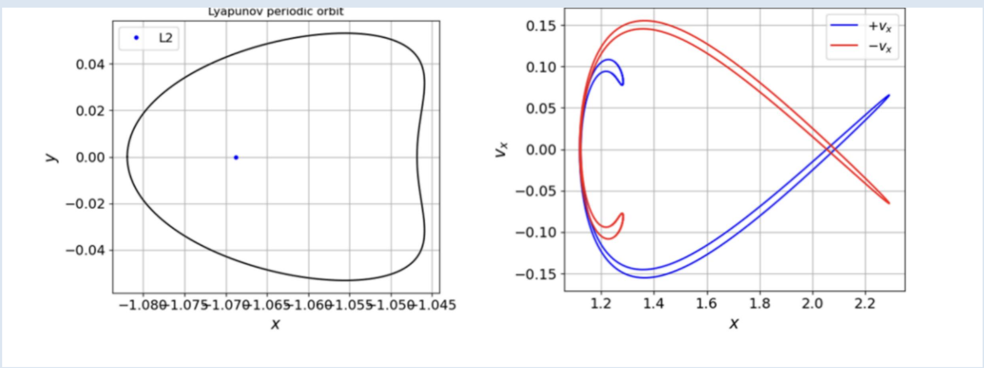
Repeated scattering can drive h2 -> h1 and unlock new resonance layers.

Findings

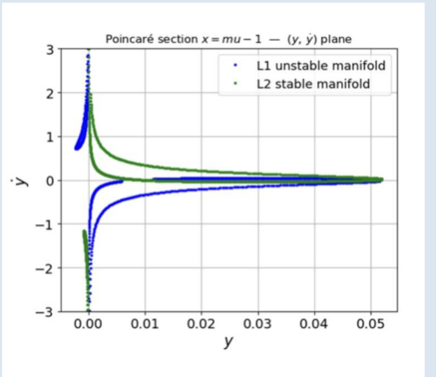
A. h1 near L1: 3:2



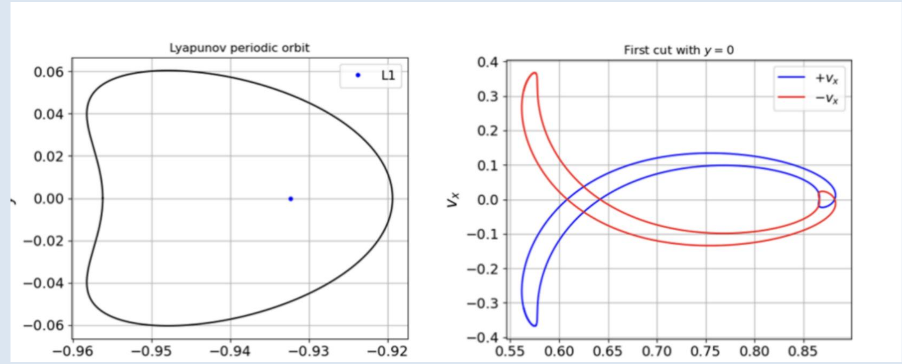
B. h1 near L2: 2:3 and 1:2



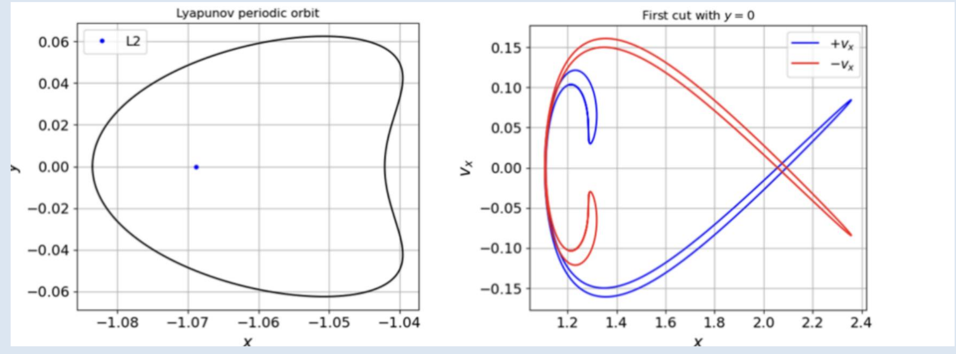
Intersection of stable and unstable manifolds for h1



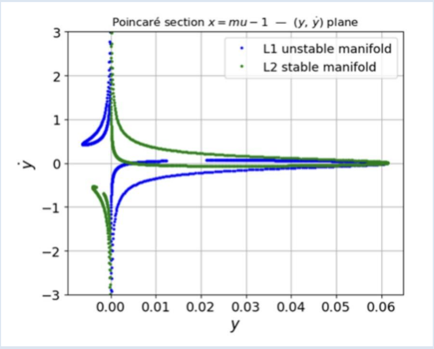
C. h2 near L1: 3:2 and 13:10



D. h2 near L2: 2:3 and 1:2



Intersection of stable and Unstable manifolds for h2



Key numerical outcomes

- At h1, the L1 family reproduces the classical 3:2 resonance, while the L2 family exhibits 2:3 and an additional 1:2 resonance.
- The 1:2 resonance corresponds to an orbit at about 8.3 AU from the Sun - further from Jupiter (~5.2 AU) and closer to Saturn (~9.5 AU) - so it provides a plausible bridge toward the later Saturn-adjacent behavior.
- At h2, the L1 family still contains 3:2 but also reveals a new 13:10 resonance. Near L2, the 2:3 and 1:2 resonances persist.
- The stable/unstable manifold geometry supports transfers between the L1 and L2 families across [h1, h2], while the elliptic perturbation supplies the energy drift needed to move between resonance layers.

Conclusions

- PCRTBP still explains the classical 3:2 <-> 2:3 exchange, but it does not capture Oterma's full later evolution on its own.
- The additional 1:2 and 13:10 resonances enlarge the transition network available to the comet.
- In PERTBP, repeated scattering can raise h1 to h2 and supply the energy drift needed for newer resonance changes.

Next steps

- Finding the normally hyperbolic invariant manifolds
- (NHIMs) and setting up the scattering map.
- Showing that repeated application of the scattering map over a long time can increase the energy from h1 to h2 (Arnold Diffusion).
- Exploring energy levels there than h2.

References

[1] Shane Ross. The interplanetary transport network. American Scientist, 94:230, 05. 2006.

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[3] Koon, W. S., Lo, M. W., Marsden, J. E., & Ross, S. D. (2000). Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics. Chaos, 10(2), 427-469.