Sunday, November 25, 2018 1:46 PM

Binary Search (or Bisection method)

→ to solve nonlinear equations for a single variable x

We rearrange the nonlinear equation in the form

$$\boxed{f(x) = 0}$$

Finding the solution is thus equivalent to find the zeros, or roots, of $f(x)$

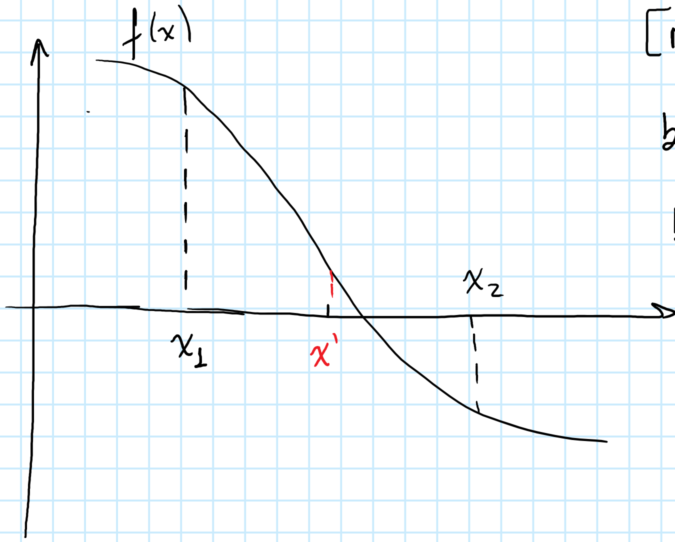
STEPS

1. we need to specify the interval $[x_1, x_2]$ in which we want to search for the solution.

   We need to choose x_1 and x_2 so that

   $$f(x_1) \cdot f(x_2) < 0,$$

   that is, either $f(x_1)$ or $f(x_2)$ is positive and the other is negative. This guarantees at least one root between x_1 and x_2.



[Note: there could also be more than 1 root, but let us assume for the moment that there is only 1.]

2 We now take the midpoint between x_1 and x_2

$$\boxed{x' = \frac{x_1 + x_2}{2}}$$

If $f(x') = 0$, this is the root.

3 •) If $f(x')$ has the same sign as $f(x_1)$ then x' and x_2 bracket the solution and will be used for step 3 with

$$x_1 = x'$$
$$x_2 = x_2$$

•) If $f(x')$ has the same sign as $f(x_2)$ then

$$x_1 = x_1$$

$x_2 = x'$

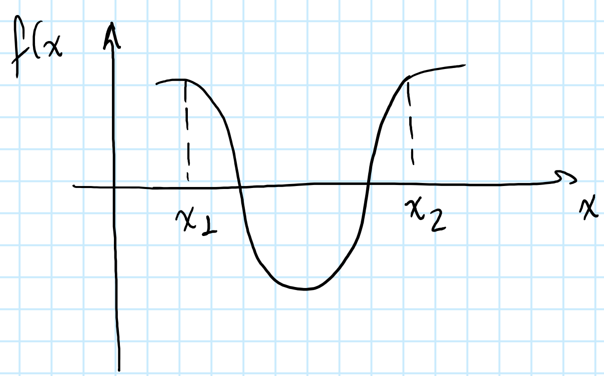
4. If $|x_1 - x_2| >$ target accuracy, repeat from (2)

Otherwise, calculate $\left(\dfrac{x_1 + x_2}{2}\right)$ once more and

this is the final estimate for the root

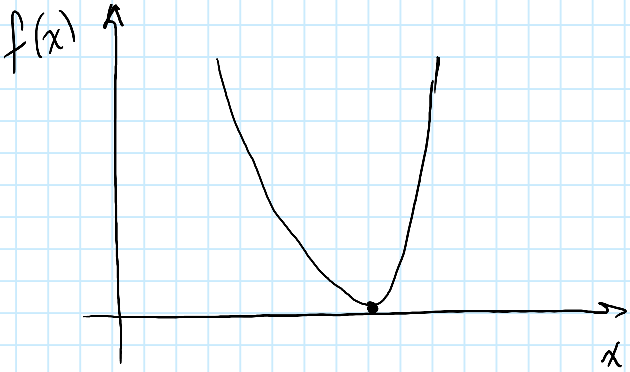
Problems with the method

→ If there are an even number of roots between x_1 and x_2, we may think there is no root.



→ It cannot find even-order polynomial roots. They just touch the horizontal axis.

so we cannot find $f(x_1) \cdot f(x_2) < 0$



Examples

$(1-x)^2 = 0$

$(2-3x)^4 = 0$

etc