Bounded Pairs in 1-D and 2-D Models

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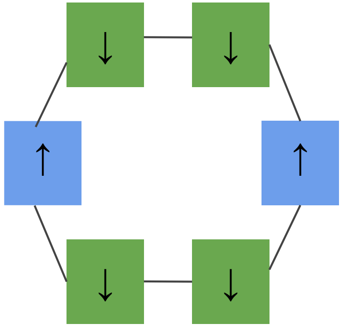
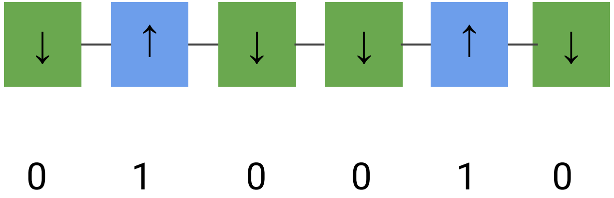
Overview...

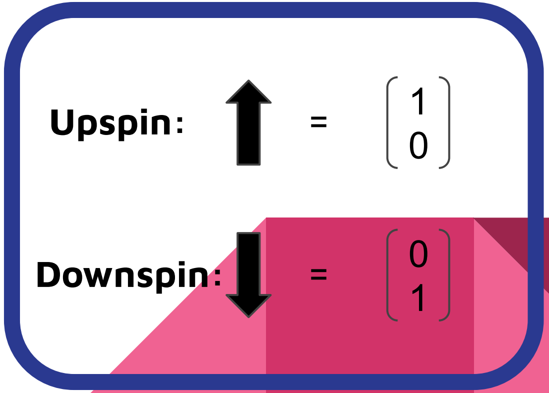
- Spin ½ model: this model describes many-body quantum systems, which are everywhere, but they are not so well understood because they are so complex.
- We studied the dynamics/evolution of these types of systems, how they depend on the interactions between the spins and on the presence of a defect within the system.
- We looked at 1D systems/chains, my colleague and friend will present the 2D case.

Our 1-D System

- We have a chain, where on each site there is a spin-½ particles. A magnetic field pointing down in the z-direction acts on the entire system, so each spin-1/2 either points up or down in the z-direction. When the spin points upwards, we call it an excitation. This spin has more energy than the down-spin, because it is against the magnetic field. A spin-½ pointing downwards is in its ground state.
- This 1D spin-½ system can be either in chain or ring formation. We consider the latter.







The Hamiltonian that describes our system

$$H = \frac{1}{2}\epsilon_d \sigma_d^z + \frac{1}{2}\sum_{j=1}^{L} h\sigma_j^z + \frac{1}{4}\sum_{j=1}^{L}\left[J_z \sigma_j^z \sigma_{j+1}^z + J_{xy}(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y)\right]$$

The Pauli spin matrices

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- All of the sigmas are the Pauli matrices
- *h* is the amplitude of the magnetic field acting on the entire chain
- ε_d is the amplitude of a magnetic field that acts on a single site and because of this is the "defect" site
- σ^z is the Pauli matrix in the z-direction. The $\sigma^z_j . \sigma^z_{j+1}$ term is called the Ising Interaction. Jz is the interaction strength of the Ising Interaction. The interaction happens between neighboring sites and affects the energy of the spin configurations.

<1010|H|1010> = -Jz

Ising Interaction

$$\frac{J_z}{4}\sigma_n^z \sigma_{n+1}^z |\uparrow_n \uparrow_{n+1}\rangle = +\frac{J_z}{4}|\uparrow_n \uparrow_{n+1}\rangle$$

$$\frac{J_z}{4}\sigma_n^z \sigma_{n+1}^z |\downarrow_n \downarrow_{n+1}\rangle = +\frac{J_z}{4}|\downarrow_n \downarrow_{n+1}\rangle$$

$$\frac{J_z}{4}\sigma_n^z \sigma_{n+1}^z |\uparrow_n \downarrow_{n+1}\rangle = -\frac{J_z}{4}|\uparrow_n \downarrow_{n+1}\rangle$$

$$\frac{J_z}{4}\sigma_n^z \sigma_{n+1}^z |\downarrow_n \uparrow_{n+1}\rangle = -\frac{J_z}{4}|\downarrow_n \uparrow_{n+1}\rangle$$

Further break down...

$$\frac{Jxy}{4}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y)|\uparrow_n\downarrow_{n+1}\rangle = +\frac{Jxy}{2}|\downarrow_n\uparrow_{n+1}\rangle$$

$$\frac{Jxy}{4}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y)|\downarrow_n\uparrow_{n+1}\rangle = +\frac{Jxy}{2}|\uparrow_n\downarrow_{n+1}\rangle$$

$$\frac{Jxy}{4}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y)|\uparrow_n\uparrow_{n+1}\rangle = 0$$

$$\frac{Jxy}{4}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y)|\downarrow_n\downarrow_{n+1}\rangle = 0$$

The J_{xy} term used in equations is also called "the flip flop" term because it reverses the direction of a pair of neighboring spins if they point in opposite directions. If the neighboring sites have spins that are in the same direction, then the J_{xy} term does not contribute at all. This means that the number of up-spins and down-spins in the chain are conserved. The Ising interaction contributes the diagonal elements of the Hamiltonian matrix (Ex: -Jz), while the flip flop term gives the off diagonal elements (Ex: $\langle 1010|H|1100\rangle = J_{xy}/2$).

	1,1,0,0	1,0,1,0	1,0,0,1	0,1,1,0	0,1,0,1	0,0,1,1
1,1,0,0	$H_{11} = \varepsilon/2$	$H_{12} = \frac{J_{xy}}{2}$	$H_{13} = 0$	$H_{14} = 0$	$H_{15} = \frac{J_{xy}}{2}$	$H_{16} = 0$
1,0,1,0	$H_{21} = \frac{J_{xy}}{2}$	$H_{22} = -J_z + \varepsilon/2$	$H_{23} = \frac{J_{xy}}{2}$	$H_{24} = \frac{J_{xy}}{2}$	$H_{25} = 0$	$H_{26} = \frac{J_{xy}}{2}$
1,0,0,1	$H_{31} = 0$	$H_{32} = \frac{J_{xy}}{2}$	$H_{33} = \varepsilon/2$	$H_{34} = 0$	$H_{35} = \frac{J_{xy}}{2}$	$H_{36} = 0$
0,1,1,0	$H_{41} = 0$	$H_{42} = \frac{J_{xy}}{2}$	$H_{43} = 0$	$H_{44} = -\varepsilon/2$	$H_{45} = \frac{J_{xy}}{2}$	$H_{46} = 0$
0,1,0,1	$H_{51} = \frac{J_{xy}}{2}$	$H_{52} = 0$	$H_{53} = \frac{J_{xy}}{2}$	$H_{54} = \frac{J_{xy}}{2}$	$H_{55} = -J_z - \varepsilon/2$	$H_{56} = \frac{J_{xy}}{2}$
0,0,1,1	$H_{61} = 0$	$H_{62} = \frac{J_{xy}}{2}$	$H_{63} = 0$	$H_{64} = 0$	$H_{56} = \frac{J_{xy}}{2}$	$H_{66} = -\varepsilon/2$

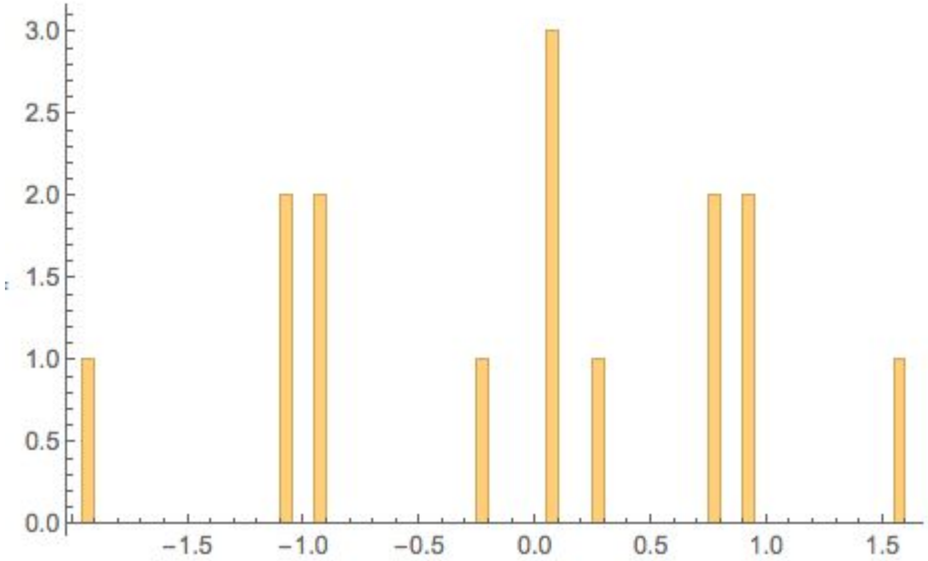
Contributions to Eigenstates from Basis Vectors

$J_z \sim J_{xy}$		$J_z >> J_{xy}$		
-0.135591	110000	0.408248	110000	0
0.29342	101000	0	101000	-0.408248
-0.352665	100100	0	100100	0
0.29342	100010	0	100010	-0.408248
-0.135591	100001	-0.408248	100001	0
-0.135591	011000	-0.408248	011000	0
0.29342	010100	0	010100	0.408248
-0.352665	010010	0	010010	0
0.29342	010001	0	010001	0.408248
-0.135591	001100	0.408248	001100	0
0.29342	001010	0	001010	-0.408248
-0.352665	001001	0	001001	0
-0.135591	000110	-0.408248	000110	0
0.29342	000101	0	000101	0.408248
-0.135591	000011	0.408248	000011	0

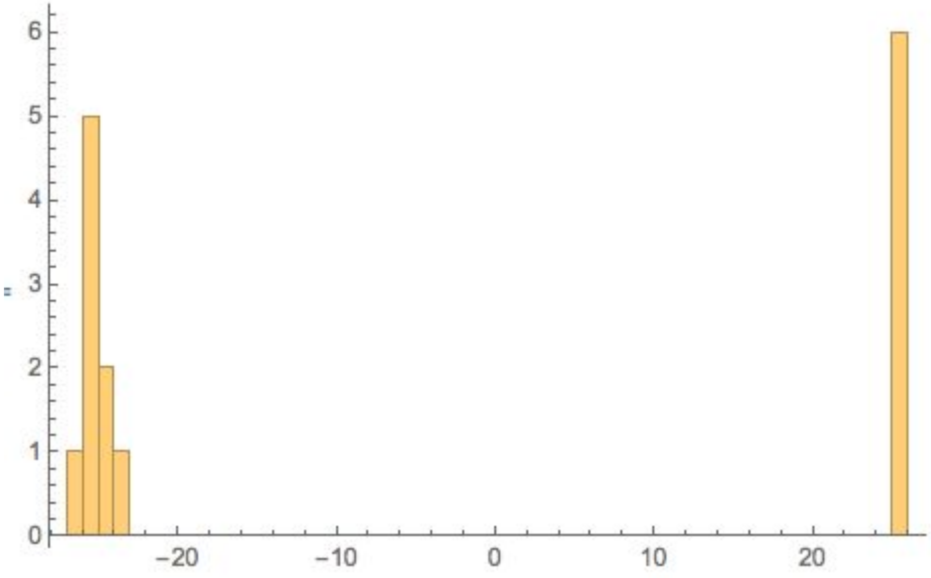
All basis vectors contribute similarly to the eigenstates

The eigenstates either have only contributions from basis vectors where the up-spins are neighbors (bound pairs) or when they are separated

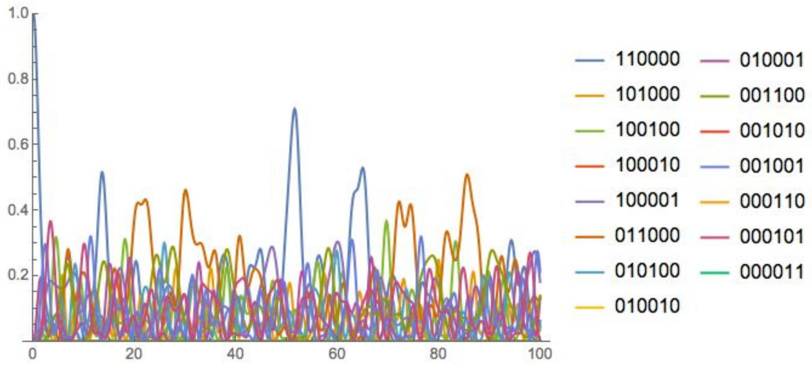
$J_z \sim J_{xy}$:



$J_z >> J_{xy}$:

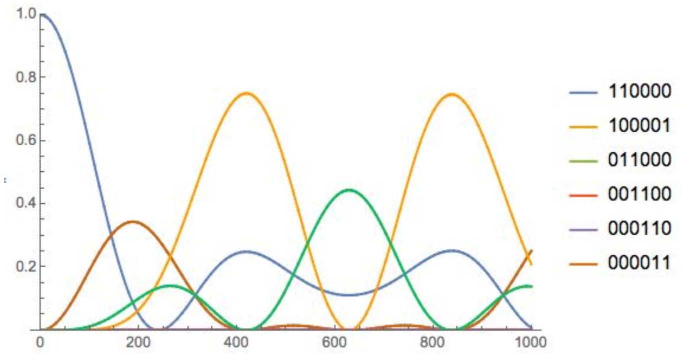


Evolution of the Initial State |110000>



$J_z \sim J_{xy}$:

In this scenario, the bound pair splits, so the excitations spread quickly throughout the chain

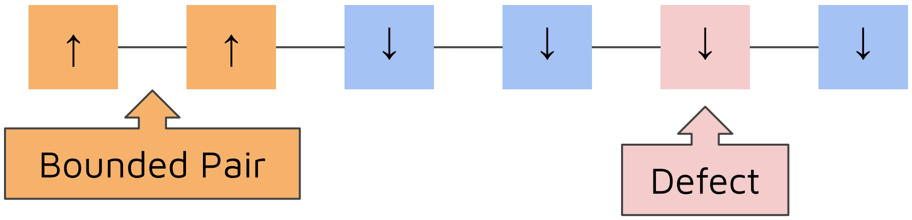


$J_z >> J_{xy}$:

In this scenario, the bound pair doesn't split, but rather it moves slowly as a single heavy excitation

System with a Defect:

Initial state 110000 where the bounded pair of the up-spins are not on the defect



Energy of the basis vectors:

$|110000\rangle = J_z/2$

$|100010\rangle = -J_z/2$

But if we add a defect on site 5:

$|110000\rangle = J_z/2 - \varepsilon_5/2$

$|100010\rangle = -J_z/2 + \varepsilon_5/2$

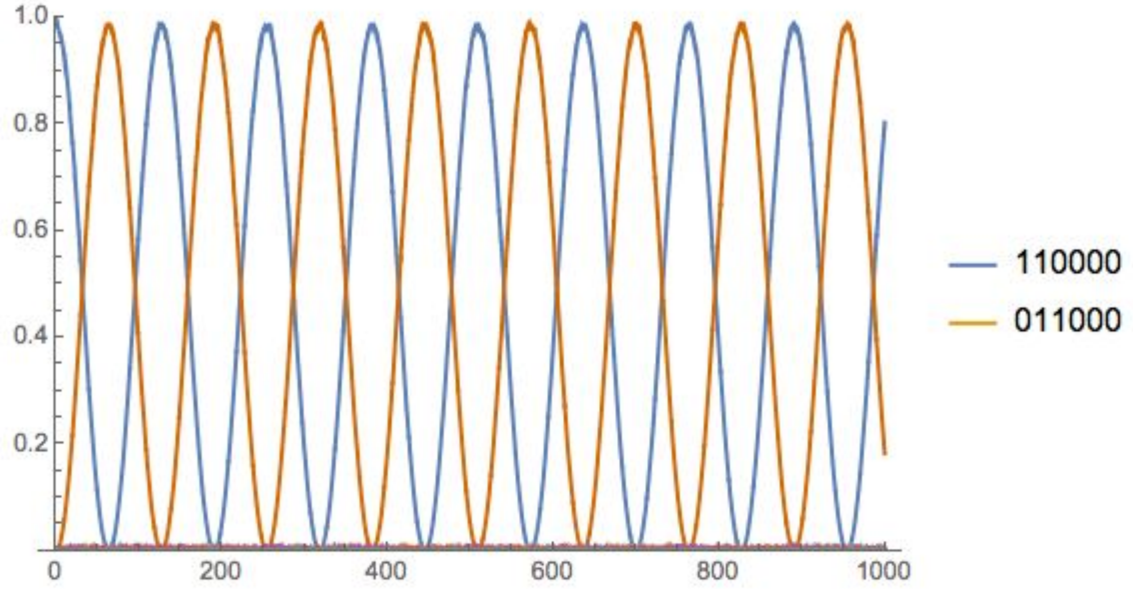
We can force the two to have the same energy by choosing

$J_z/2 - \varepsilon_5/2 == -J_z/2 + \varepsilon_5/2$

That is, ε_5 = Jz, so that a bound pair away from the defect has energy=0

An unbounded pair with one excitation on the defect also has energy =0

Evolution of |110000> with a Defect:



Bounded pairs are robust - we cannot split them!