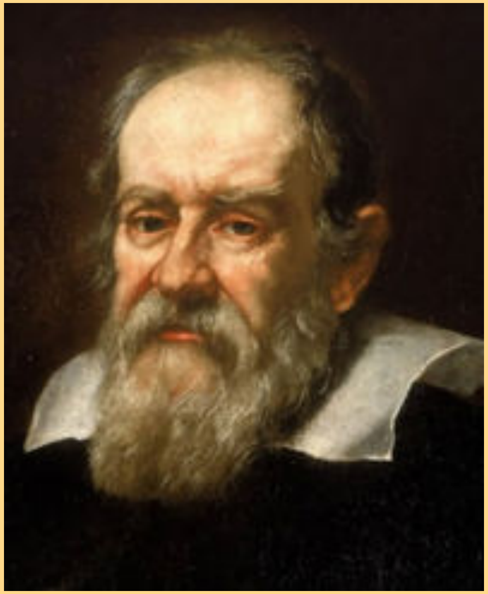
Chapter 2

Mechanics – study the motion of objects
(**classical mechanics)**



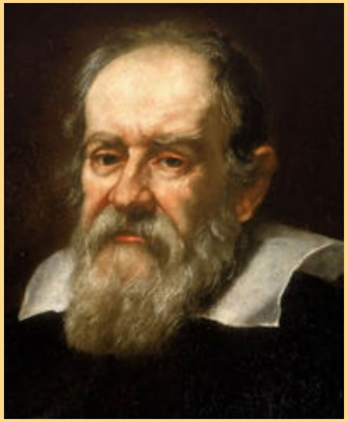
Galileo Galilei (1564-1642)
Uniformly accelerated motions
Telescope, astronomical observations
Copernican theory

Isaac Newton (1642-1727)
Universal gravitation
Laws of motion, calculus (Liebniz)
Optics

“Eppur se muove”
Bertold Brecht – *Galileo*

Dava Sobel
Galileo’s Daugther
Longitude

Chapter 2



Galileo Galilei (1564-1642)

Émilie du Châtelet (1706-1749)

Isaac Newton (1642-1727)

Kinematics in One Dimension

Mechanics – study the motion of objects
(**classical mechanics)**

Kinematics – how objects move
Dynamics – deals with forces and why objects move the way they do

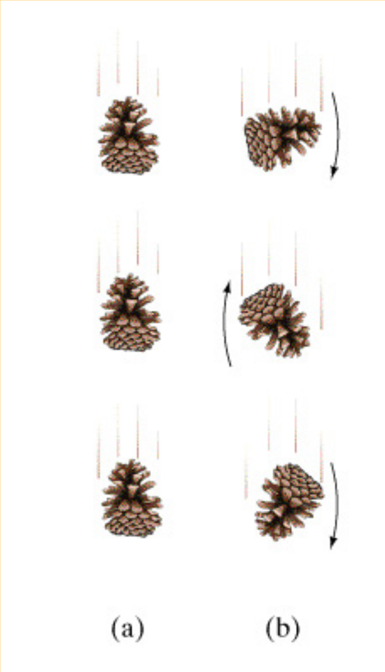
Kinematics

Translational motion [as in (a)]
no rotation [as in (b)]

One-dimensional translational motion (straight line)

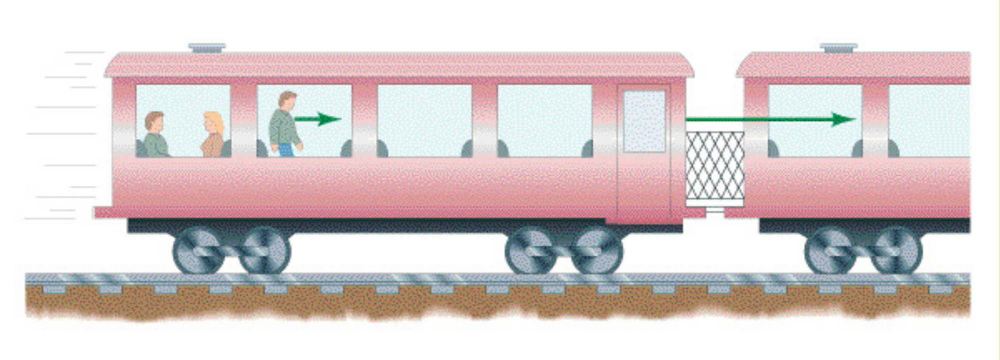
Idealized **particle**
(mathematical point with no spatial extent)

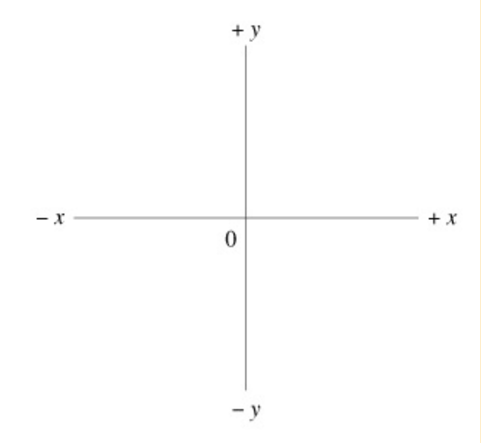
Idealizations, simplifications are common in modern science



Reference Frames

Any measurement of position, distance, or speed is made with respect to a **reference frame** or **frame of reference**





To specify the **translational** motion of an object one needs:

position - set of **coordinate axes**
speed
direction

One-dimensional motion: **position** given by x coordinate (horizontal motion)
or y coordinate (vertical motion)

Distance vs. Displacement

Distance:
how much the object traveled

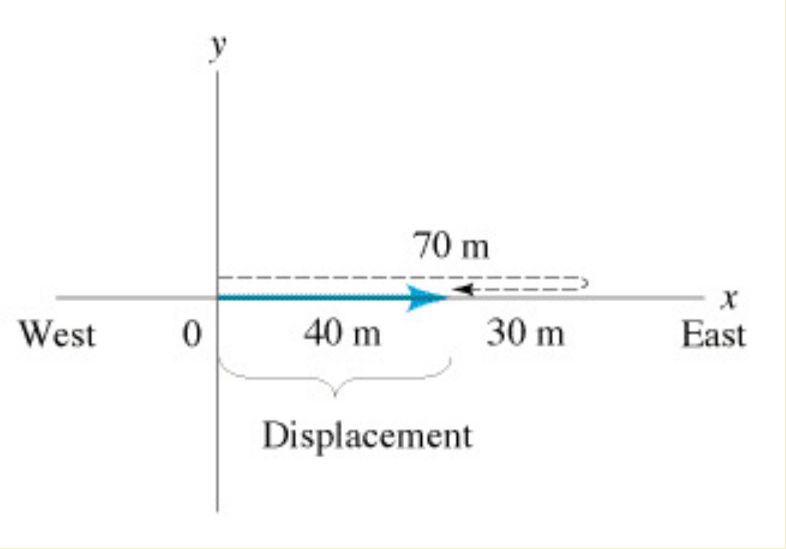
It is a SCALAR (=number) with units

Its value is always positive

Displacement:
how far the object is from its starting point

has magnitude and direction - **VECTOR**

In one dimension, its direction is defined by a sign (+ or -)



$$\boxed{\Delta x = x_2 - x_1}$$

x_2 - final position

x_1 - initial position

Speed and Velocity

Average speed: total **distance** divided by time elapsed
(positive NUMBER- SCALAR with units)

$$\boxed{average\ speed = \frac{distance}{\Delta t}}$$

Average velocity: total **displacement** divided by time elapsed
(**VECTOR** – has magnitude and direction)

$$\boxed{\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}}$$

Direction of the average velocity = direction of displacement

Units: m/s

Example: 70 m east and 30 m west, time elapsed=10 s

Average speed=10m/s; magnitude of the average velocity=4m/s

Average speed and average velocity have the same magnitude if the motion is all in one direction

Average Velocity - exercises

1) During a 3.00s time interval, a runner's position changes from 50.0m to 30.5m. What is the runner's average velocity?

2) How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18km/h

3) A boat can move at 30 km/h in still water. How long will it take to move 12 km upstream in a river flowing 6.0 km/h?

1) – 6.50 m/s

2) 45 km

3) 30 min

Instantaneous Velocity

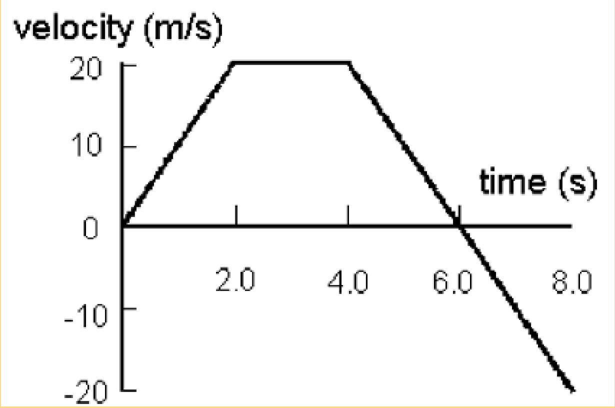
Instantaneous velocity = the average velocity during an infinitesimal short time (VECTOR)

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}$$

derivative

Instantaneous speed = magnitude of the instantaneous velocity

Object at **UNIFORM** (=constant) velocity then
instantaneous velocity=average velocity



In the book:
Velocity=instantaneous velocity
vs.
Average velocity

Acceleration

Average acceleration = change in velocity divided by the elapsed time
it is a VECTOR

$$\boxed{\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}}$$

Instantaneous acceleration

$$\boxed{a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}}$$

ATTENTION!
direction of acceleration is
not necessarily the same as the
direction of displacement

Acceleration tells how quickly the velocity changes
Velocity tells how quickly the position changes

Units: m/s^2

Deceleration ***DOES NOT NECESSARILY*** mean that the acceleration is negative

Acceleration - exercises

1) A car accelerates along a straight line from rest to 36km/h in 5.0s. What is magnitude of its average acceleration? (Careful with units)
2) (a) If the acceleration is zero, does it mean that the velocity is zero? (b) If the velocity is zero, does it mean that the acceleration is zero?
3) A car is moving in the positive direction. The driver puts on the brakes. If the initial velocity is 15m/s and it takes 5.0 s to slow down to 5.0m/s, what is the car's average acceleration
4) Same as (3), but the car is moving in the negative direction.

1) 2.0 m/s^2

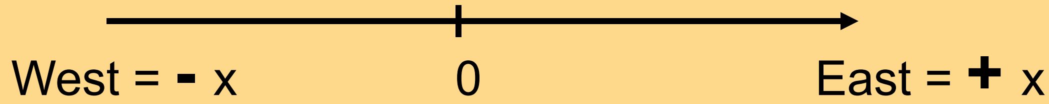
2) (a) and (b) Not necessarily

3) - 2.0 m/s^2

4) + 2.0 m/s^2

Uniform velocity

In one dimension: direction is + or -



Vector $\longrightarrow$ has positive direction

Vector $\longleftarrow$ has negative direction

The magnitude is the absolute value

$$\boxed{\bar{v} = \frac{\Delta x}{\Delta t} = \frac{displacement}{elapsed\ time}}$$

Direction of the average velocity = direction of displacement

Uniform velocity:
constant $v = \bar{v}$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} \Rightarrow v = \frac{x - x_0}{t}$$

$$x - x_0 = vt \Rightarrow \boxed{x = x_0 + vt}$$

Direction of the acceleration

In one dimension: direction is + or -

West = **-** x 0 East = **+** x

Vector → has positive direction

Vector ← has negative direction

The magnitude is the absolute value

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{\textit{change in velocity}}{\textit{elapsed time}}$$

*Magnitude of **velocity is increasing**:*

Direction of the average acceleration **=** *direction of the velocity (displacement)*

*Magnitude of **velocity is DEcreasing**:*
Direction of the average acceleration
is opposite to the
direction of the velocity (displacement)

Uniform acceleration: $a = \bar{a}$
(constant)

BUT velocity is NOT constant!! $v \neq \bar{v}$

Constant Acceleration

Magnitude of acceleration is constant: instantaneous and average acceleration are equal

$$\bar{a} = a; \quad a = \frac{v - v_0}{t};$$

$$\Rightarrow \boxed{v = v_0 + at}$$

(Linear in time)

(average velocity =midway)

$$\boxed{\bar{v} = \frac{v_0 + v}{2}}$$

$$\bar{v} = \frac{x - x_0}{t}; \quad \bar{v} = \frac{v_0 + v}{2}; \quad v = v_0 + at$$

$$\Rightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$

(Quadratic in time)

$$x = x_0 + \bar{v} t; \quad \bar{v} = \frac{v_0 + v}{2}; \quad v = v_0 + at$$

$$\Rightarrow \boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

(useful when time is unknown)

Classical mechanics: give us the initial conditions and we can **predict** the motion of any particle

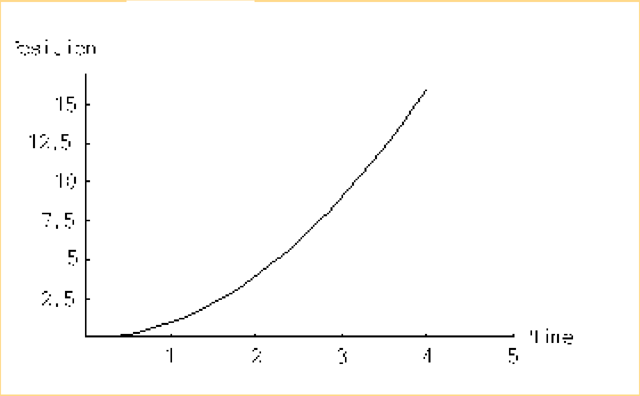
Linear vs. Quadratic

$$v = v_0 + at$$

$$v_0 = 0; \quad a = 2m/s^2$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$x_0 = 0 \quad v_0 = 0 \quad a = 2m/s^2$$



Constant Acceleration - exercises

1. How long does it take a car to cross a 36.0 m wide intersection after the light turns green, if the car accelerates from rest at a constant acceleration of 2.00 m/s^2

2. Design an airport for small planes. One kind of airplane that uses this airfield must reach a speed before takeoff of at least 30.0 m/s and can accelerate at 4.50 m/s^2. (a) If the runway is 81 m long, can this plane reach the required speed for take off? (b) If not, what minimum length must the runway have?

1) t=6.00 s

2) (a) No, because it only reaches 27m/s
(b) 100 m

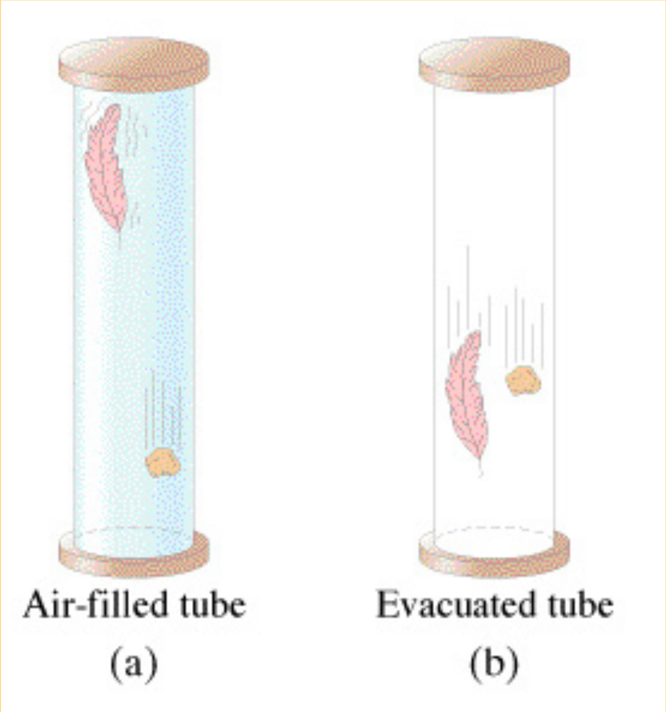
Read Example 2-9 in your book

Falling Objects

Free fall – uniformly accelerated motion (constant acceleration)

Galileo: Objects increase their speed as they fall

$\boxed{d \propto t^2}$ and experiments to support it



Air acts as a resistance to very light objects with a large surface,
but in the absence of resistance
all objects fall with the ***same constant acceleration***

Same constant acceleration

Free fall – uniformly accelerated motion (**constant** acceleration)

Heavy objects **DO NOT** fall faster than lighter objects
(mass does not appear in the equations)

All objects fall with the ***same constant acceleration***
in the absence of air or any other resistance

Galileo is the father of **modern science** not only for the content of his science but also for his approach:
idealization, simplification, theory, experiments

Acceleration due to gravity

Acceleration due to gravity is a **VECTOR** and its direction is toward the center of Earth

$$\downarrow \quad \boxed{g = 9.80 m/s^2}$$

Here, we neglect the effects of air resistance

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow y = y_0 + v_0 t + \frac{1}{2} g t^2$$

$$v = v_0 + gt$$

$$v^2 = v_0^2 + 2g(y - y_0)$$

$$\bar{v} = \frac{v + v_0}{2}$$

It is arbitrary to choose y positive in the upward or downward direction; but we must be consistent

Free Fall - exercises

g=9.80 m/s^2

1. A ball is dropped from a tower 70.0 m high. How far will the ball have fallen after 1.00s and after 2.00s? What will be the magnitude of its velocity at these points?

2. Now consider the same exercise, but suppose the ball is thrown downward with an initial velocity of 3.00m/s

3. A person throws a ball upward with initial velocity 15.0m/s. Calculate
(a) how high it goes, (b) how long it takes to reach the maximum height,
(c) how long the ball is in the air before it comes back to his hand again,
(d) the velocity of the ball when it returns to the thrower's hand,
(e) the time the ball passes a point 8.00 m above the person's hand.

1) 1s fell 4.9 m; 2s fell 19.6 m; v(1s)=9.80 m/s; v(2s)=19.6 m/s

2) 1s fell 7.90 m; 2s fell 25.6 m; v(1s)=12.8 m/s; v(2s)=22.6 m/s

3) (a) 11.5 m; (b) 1.53s; (c) 3.06s; (d) – 15.0 m/s; (e) 0.16s and 2.37s

Free Fall and Frames

Let us solve Problem 47 together: (discuss reference frame)
A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff 70.0 m high.
(a)How much later does it reach the bottom of the cliff? ~~-2.7s~~ 5.2s
(b)What is its speed just before hitting? 38.9 m/s
(c)What total distance did it travel? 84.7 m

0 at the top of the cliff ***(more convenient to choose 0 where the motion starts)***
70 m at the bottom of the cliff
initial velocity upward is negative
g is positive

0 at the bottom of the cliff
70 m at the edge of the cliff
g is negative
initial velocity upward is positive

0 at the top of the cliff
--70 m at the edge of the cliff
g is negative
initial velocity upward is positive

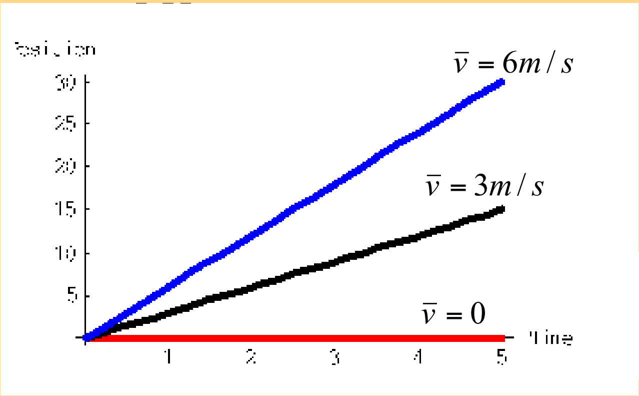
In both frames: magnitudes are the same, but directions may change

Graphical Analysis of Linear Motion

The axes of any graph must have units

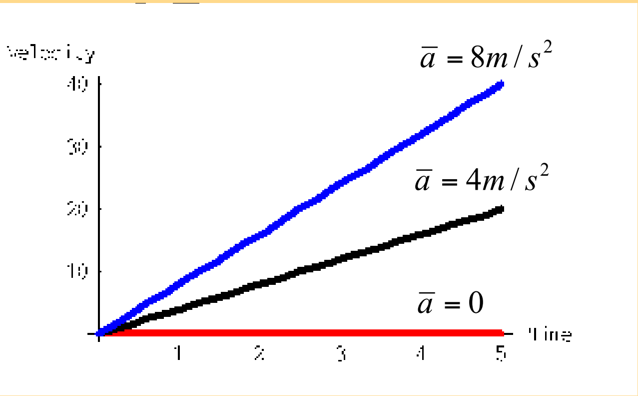
Slope of the graph of position vs. time gives the velocity

uniform velocity

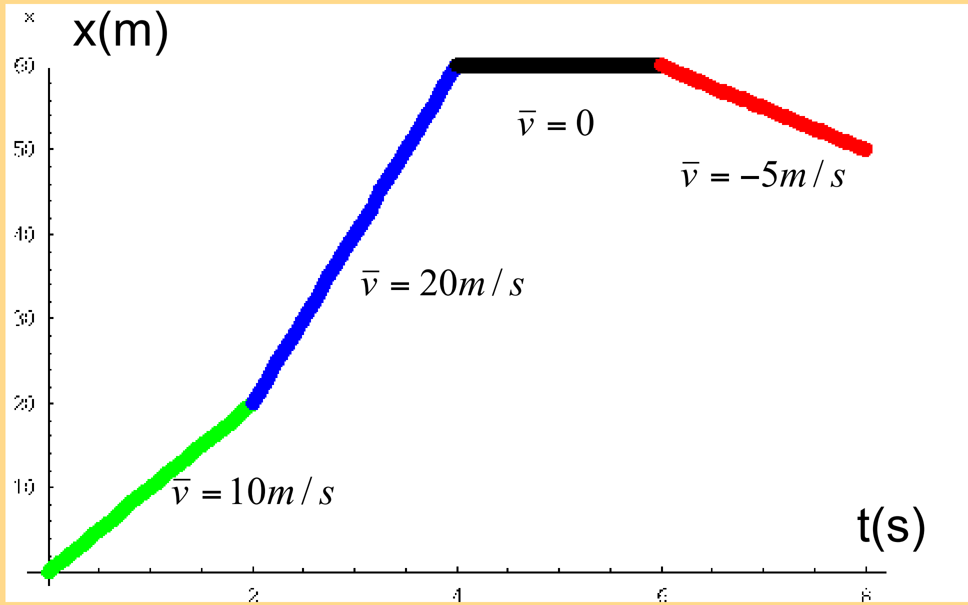


Slope of the graph of velocity vs. time gives the acceleration

uniform acceleration



Constant Velocity



$$\boxed{x = x_0 + \bar{v}t}$$

$x = 0 + 10t \qquad 0 \ to \ 2s$

$x = 20 + 20(t-2) \qquad 2s \ to \ 4s$

$x = 60 \qquad 4s \ to \ 6s$

$x = 60 - 5(t-6) \qquad 6s \ to \ 8s$

Acceleration is zero throughout

Green: object moves with constant velocity in the positive direction

Blue: object moves with constant velocity in the positive direction

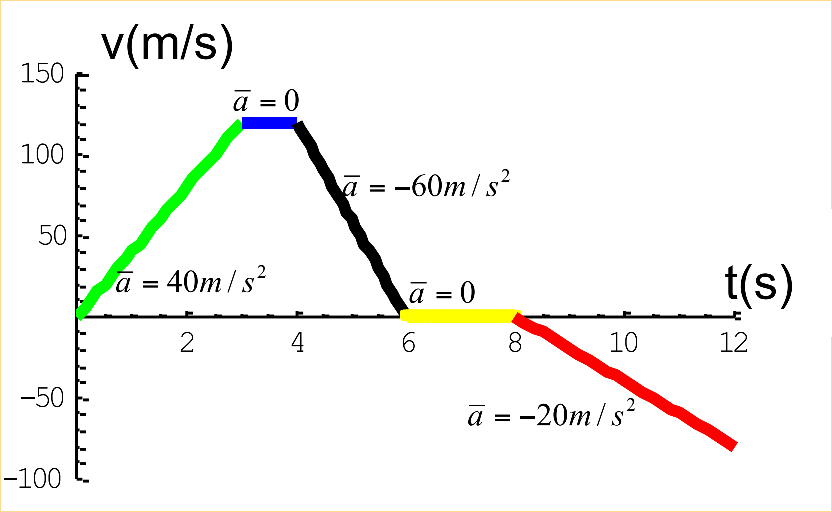
Here, the velocity is greatest, because the slope is the highest

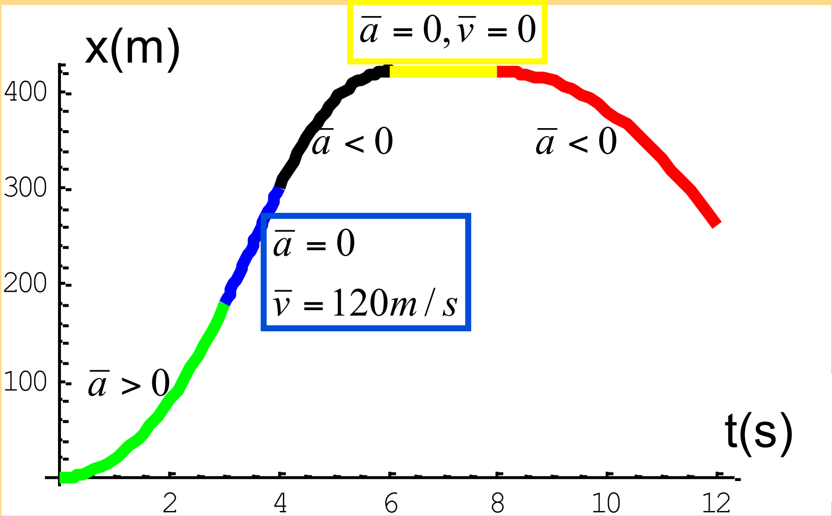
Black: the object stopped from 4 s to 6 s

Red: the object moves with constant velocity in the negative direction

At 8 s: displacement=50m, total distance traveled=70m

Constant Acceleration





Green: + displacement, + velocity
magnitude of velocity increases, + acceleration, constant acceleration

Blue: + displacement, + velocity
constant velocity, acceleration=0

Black: + displacement, + velocity
magnitude of velocity **decreases, - acceleration** , constant acceleration

Yellow: the object stops

Red: - displacement, - velocity
magnitude of velocity **increases, - acceleration,** constant acceleration

Black: largest magnitude of the acceleration – highest slope
Maximum magnitude of the velocity is 120m/s