Chapter 2

Mechanics – study the motion of objects (classical mechanics)

Galileo Galilei (1564-1642)
- Uniformly accelerated motions
- Telescope, astronomical observations
- Copernican theory

Isaac Newton (1642-1727)
- Universal gravitation
- Laws of motion, calculus (Liebniz)
- Optics

“Eppur se muove”
Bertold Brecht – Galileo

Dava Sobel
Galileo’s Daughter
Longitude
Chapter 2

Galileo Galilei (1564-1642)

Émilie du Châtelet (1706-1749)

Isaac Newton (1642-1727)
Kinematics in One Dimension

Mechanics – study the motion of objects (classical mechanics)

Kinematics – how objects move
Dynamics – deals with forces and why objects move the way they do

Kinematics

**Translational** motion [as in (a)]
no rotation [as in (b)]

**One-dimensional** translational motion (straight line)

Idealized particle
(mathematical point with no spatial extent)

Idealizations, simplifications are common in modern science
Any measurement of position, distance, or speed is made with respect to a **reference frame** or **frame of reference**.

To specify the **translational** motion of an object one needs:
- position - set of **coordinate axes**
- speed
- direction

One-dimensional motion: **position** given by x coordinate (horizontal motion) or y coordinate (vertical motion).
Distance vs. Displacement

**Distance:**
- how much the object traveled
- It is a SCALAR (=number) with units
- Its value is always positive

**Displacement:**
- how far the object is from its starting point
- has magnitude and direction - VECTOR
- In one dimension, its direction is defined by a sign (+ or -)

\[ \Delta x = x_2 - x_1 \]

- \( x_2 \) - final position
- \( x_1 \) - initial position
Speed and Velocity

**Average speed:** total distance divided by time elapsed  
(positive NUMBER - SCALAR with units)

\[
\text{average speed} = \frac{\text{distance}}{\Delta t}
\]

**Average velocity:** total displacement divided by time elapsed  
(VECTOR – has magnitude and direction)

\[
\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}
\]

*Direction of the average velocity = direction of displacement*

**Units:** m/s

Example: 70 m east and 30 m west, time elapsed=10 s  
Average speed=10 m/s; magnitude of the average velocity=4 m/s

Average speed and average velocity have the same magnitude if the motion is all in one direction.
1) During a 3.00s time interval, a runner’s position changes from 50.0m to 30.5m. What is the runner’s average velocity?

2) How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18km/h?

3) A boat can move at 30 km/h in still water. How long will it take to move 12 km upstream in a river flowing 6.0 km/h?

1) – 6.50 m/s

2) 45 km

3) 30 min
Instantaneous velocity = the average velocity during an infinitesimal short time

\[ v = \lim_{{\Delta t \to 0}} \frac{\Delta x}{\Delta t} \]

forgettable

Instantaneous speed = magnitude of the instantaneous velocity

Object at **UNIFORM** (constant) velocity then

instantaneous velocity = average velocity

In the book:

Velocity = instantaneous velocity vs.
Average velocity
Acceleration

Average acceleration = change in velocity divided by the elapsed time
it is a VECTOR
\[ \bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \]

Instantaneous acceleration
\[ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \]

Acceleration tells how quickly the velocity changes
Velocity tells how quickly the position changes

Units: \( m / s^2 \)

Deceleration DOES NOT NECESSARILY mean that the acceleration is negative
1) A car accelerates along a straight line from rest to 36km/h in 5.0s. What is magnitude of its average acceleration? (Careful with units)

2) (a) If the acceleration is zero, does it mean that the velocity is zero? (b) If the velocity is zero, does it mean that the acceleration is zero?

3) A car is moving in the positive direction. The driver puts on the brakes. If the initial velocity is 15m/s and it takes 5.0 s to slow down to 5.0m/s, what is the car’s average acceleration

4) Same as (3), but the car is moving in the negative direction.

1) 2.0 \text{ m/s}^2
2) (a) and (b) Not necessarily
3) - 2.0 \text{ m/s}^2
4) + 2.0 \text{ m/s}^2
Uniform velocity

In one dimension: direction is + or -

West = - x 0
East = + x

Vector → has positive direction
Vector ← has negative direction

\[ \vec{v} = \frac{\Delta x}{\Delta t} = \frac{\text{displacement}}{\text{elapsed time}} \]

Direction of the average velocity = direction of displacement

Uniform velocity: constant \( v = \bar{v} \)

\[ \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} \Rightarrow v = \frac{x - x_0}{t} \]

\[ x - x_0 = vt \Rightarrow x = x_0 + vt \]
Direction of the acceleration

In one dimension: direction is + or -

West = - x

East = + x

Vector has positive direction

Vector has negative direction

The magnitude is the absolute value

\[ \bar{a} = \frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{elapsed time}} \]

Magnitude of \textit{velocity is increasing}:

Direction of the average acceleration =
direction of the velocity (displacement)

Magnitude of \textit{velocity is DEcreasing}:

Direction of the average acceleration is opposite to the
direction of the velocity (displacement)

Uniform acceleration: \( a = \bar{a} \)

BUT velocity is NOT constant!! \( v \neq \bar{v} \)
Constant Acceleration

Magnitude of acceleration is constant: instantaneous and average acceleration are equal

\[ \bar{a} = a; \quad a = \frac{v - v_0}{t}; \]

\( \Rightarrow \quad v = v_0 + at \)

(Linear in time)  (average velocity = midway)

\[ \bar{v} = \frac{v_0 + v}{2}; \quad v = v_0 + at \]

\( \Rightarrow \quad x = x_0 + v_0t + \frac{1}{2}at^2 \)

(Quadratic in time )

\[ x = x_0 + \bar{v}t; \quad \bar{v} = \frac{v_0 + v}{2}; \quad v = v_0 + at \]

\( \Rightarrow \quad v^2 = v_0^2 + 2a(x - x_0) \)

Classical mechanics: give us the initial conditions and we can predict the motion of any particle
Linear vs. Quadratic

\[ v = v_0 + at \]
\[ v_0 = 0; \quad a = 2m/s^2 \]

\[ x = x_0 + v_0t + \frac{1}{2}at^2 \]
\[ x_0 = 0 \quad v_0 = 0 \quad a = 2m/s^2 \]
1. How long does it take a car to cross a 36.0 m wide intersection after the light turns green, if the car accelerates from rest at a constant acceleration of \(2.00 \ m/s^2\)

2. Design an airport for small planes. One kind of airplane that uses this airfield must reach a speed before takeoff of at least 30.0 m/s and can accelerate at \(4.50 \ m/s^2\). (a) If the runway is 81 m long, can this plane reach the required speed for take off? (b) If not, what minimum length must the runway have?

1) \(t=6.00\) s

2) (a) No, because it only reaches 27m/s
   (b) 100 m

Read Example 2-9 in your book
Falling Objects

**Free fall** – uniformly accelerated motion (constant acceleration)

**Galileo:** Objects increase their speed as they fall

\[ d \propto t^2 \]

and experiments to support it

Air acts as a resistance to very light objects with a large surface, but in the absence of resistance

*all* objects fall with the *same constant acceleration*
Same constant acceleration

Free fall - uniformly accelerated motion (constant acceleration)

Heavy objects **DO NOT** fall faster than lighter objects
(mass does not appear in the equations)

*All* objects fall with the *same constant acceleration*
in the absence of air or any other resistance

Galileo is the father of **modern science** not only for the content of his science but also for his approach:

- idealization, simplification, theory, experiments
Acceleration due to gravity is a **VECTOR** and its direction is toward the center of Earth

\[ g = 9.80 \text{ m/s}^2 \]

Here, we neglect the effects of air resistance

\[
\begin{align*}
    x &= x_0 + v_0 t + \frac{1}{2} at^2 \\
    y &= y_0 + v_0 t + \frac{1}{2} gt^2 \\
    v &= v_0 + gt \\
    v^2 &= v_0^2 + 2g(y - y_0) \\
    \bar{v} &= \frac{v + v_0}{2}
\end{align*}
\]

*It is arbitrary to choose y positive in the upward or downward direction; but we must be consistent*
Free Fall - exercises

g=9.80 \text{ m/s}^2

1. A ball is dropped from a tower 70.0 m high. How far will the ball have fallen after 1.00s and after 2.00s? What will be the magnitude of its velocity at these points?

2. Now consider the same exercise, but suppose the ball is thrown downward with an initial velocity of 3.00m/s

3. A person throws a ball upward with initial velocity 15.0m/s. Calculate (a) how high it goes, (b) how long it takes to reach the maximum height, (c) how long the ball is in the air before it comes back to his hand again, (d) the velocity of the ball when it returns to the thrower’s hand, (e) the time the ball passes a point 8.00 m above the person’s hand.

1) 1s fell 4.9 m; 2s fell 19.6 m; \quad v(1s)=9.80 \text{ m/s}; v(2s)=19.6 \text{ m/s}

2) 1s fell 7.90 m; 2s fell 25.6 m; \quad v(1s)=12.8 \text{ m/s}; v(2s)=22.6 \text{ m/s}

3) (a) 11.5 m; (b) 1.53s; (c) 3.06s; (d) – 15.0 m/s; (e) 0.16s and 2.37s
Let us solve Problem 47 together: (discuss reference frame)
A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff 70.0 m high.
(a) How much later does it reach the bottom of the cliff? -2.7 s 5.2 s
(b) What is its speed just before hitting? 38.9 m/s
(c) What total distance did it travel? 84.7 m

0 at the top of the cliff    (more convenient to choose 0 where the motion starts)
70 m at the bottom of the cliff
initial velocity upward is negative
$g$ is positive

0 at the top of the cliff
--70 m at the edge of the cliff
$g$ is negative
initial velocity upward is positive

In both frames: magnitudes are the same, but directions may change
The axes of any graph must have units.

Slope of the graph of position vs. time gives the velocity:

- \( v = 6 \text{ m/s} \)
- \( v = 3 \text{ m/s} \)
- \( v = 0 \)

Slope of the graph of velocity vs. time gives the acceleration:

- \( \ddot{a} = 8 \text{ m/s}^2 \)
- \( \ddot{a} = 4 \text{ m/s}^2 \)
- \( \ddot{a} = 0 \)
Acceleration is zero throughout
Green: object moves with constant velocity in the positive direction
Blue: object moves with constant velocity in the positive direction
Here, the velocity is greatest, because the slope is the highest
Black: the object stopped from 4 s to 6 s
Red: the object moves with constant velocity in the negative direction
At 8 s: displacement=50m, total distance traveled=70m
Constant Acceleration

Green: + displacement, + velocity
- magnitude of velocity increases, + acceleration, constant acceleration

Blue: + displacement, + velocity
- constant velocity, acceleration=0

Black: + displacement, + velocity
- magnitude of velocity decreases, - acceleration, constant acceleration

Yellow: the object stops

Red: - displacement, - velocity
- magnitude of velocity increases, - acceleration, constant acceleration

Black: largest magnitude of the acceleration – highest slope
Maximum magnitude of the velocity is 120m/s