Realistic Many-Body Quantum Systems vs. Full Random Matrices: Static and Dynamical Properties

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Non-equilibrium many-body quantum systems

• 1D many-body systems far from equilibrium
  • Experimentally accessible
  • Technological applications
  • Not well understood

• Goal: Understand and characterize:
  • Dynamics at different time scales
    • Dependence on chaoticity
  • Conditions for thermalization

• Use random matrices
Full Random Matrices

• Matrix filled with random numbers
  • First used by Wigner to model heavy nuclei
  • Interactions treated statistically
  • Details overlooked
  • Constrain to satisfy symmetries: real and symmetric (GOE)

• Not realistic
  • All particles interact at the same time
  • Faraway interactions as strong as nearby interactions

• Advantages
  • Analytical results
  • References and bounds for realistic systems
# Realistic Hamiltonians

<table>
<thead>
<tr>
<th>Model</th>
<th>Hamiltonian</th>
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</thead>
<tbody>
<tr>
<td><strong>Integrable XXZ model</strong></td>
<td>$H = H_{XXZ} + \epsilon_1 S^z_1$</td>
</tr>
<tr>
<td><strong>Chaotic defect model</strong></td>
<td>$H = H_{XXZ} + dJS^{z}_{[L/2]} + \epsilon_1 S^z_1$</td>
</tr>
<tr>
<td><strong>Chaotic NNN model</strong></td>
<td>$H = H_{XXZ} + \lambda H_{NNN} + \epsilon_1 S^z_1$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>1</td>
</tr>
<tr>
<td>$\epsilon_1$</td>
<td>.1</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>.48</td>
</tr>
<tr>
<td>$d$</td>
<td>.9</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1</td>
</tr>
</tbody>
</table>

$H_{XXZ} = J \sum_{n=1}^{L-1} \left( S^n_x S_{n+1}^x + S^n_y S_{n+1}^y + \Delta S^n_z S_{n+1}^z \right)$

$H_{NNN} = J \sum_{n=1}^{L-2} \left( S^n_x S_{n+2}^x + S^n_y S_{n+2}^y + \Delta S^n_z S_{n+2}^z \right)$

$S^z_1$: small defect on first site to break symmetries

d$JS^{z}_{[L/2]}$: defect in the middle
Static Properties: Level Spacing Distribution

• Full Random matrix: Wigner Dyson

\[ P(s) = \frac{\pi s}{2} \exp \left( -\frac{\pi s^2}{4} \right) \]

• Black curves: numerical data
• Red curve: Wigner Dyson
• Blue curve: Poisson
Static Properties: density of states

- Full Random matrix:

\[
\rho^{DOS}(E) = \frac{2}{\pi\varepsilon} \sqrt{1 - \left(\frac{E}{\varepsilon}\right)^2}
\]

\(-\varepsilon \leq E \leq \varepsilon\)
Static Properties: Entropies

- Shannon: $S_{sh}^\alpha = - \sum_k |C_k^\alpha|^2 \ln |C_k^\alpha|^2$
- Entanglement: $\rho_A = Tr_B(\rho)$; $S_{vN} = -Tr(\rho_A \ln \rho_A)$

![Graphs showing entropy properties](image)

- Black: Numerical Data
- Red: $\ln(0.48D)$

![Graphs showing entropy properties](image)

- Black: Normalized Entanglement Entropy
- Red: Normalized Shannon Entropy
Dynamic Properties: Survival Probability

- \( W_{ini}(t) \equiv |\langle \Psi(0) | e^{-iHt} | \Psi(0) \rangle|^2 = | \int P_{ini,ini}(E) e^{-iEt} dE |^2 \)

- LDOS: \( P_{ini,ini}(E) = \sum \alpha |C^\alpha_{ini}|^2 \delta(E - E_\alpha) \)

- Initial State: Néel state: \(|\downarrow\uparrow\downarrow\uparrow\uparrow\cdots\rangle\)

![Full Random Matrix](image1)

![Integrable XXZ](image2)

![Chaotic defect](image3)
Dynamic Properties: Survival Probability

• Full Random matrix
  • $W_{\text{int}}(t) = \frac{[J_1(2\sigma_{\text{init}})]^2}{\sigma_{\text{init}}^2 t^2}$
  • Bessel decay
  • Envelope $\sim t^{-3}$

• Realistic Hamiltonians:
  • Gaussian decay
    • Even integrable system
  • Black: Numerical data
  • Red: Gaussian
Dynamic Properties: Entropies

• $S_{sh}(t) = -\sum_{k=1}^{D} W_k(t) \ln W_k(t)$

• $W_k(t) = \left| \langle \phi_k | e^{-iHt} | \Psi(0) \rangle \right|^2$

• $S_{sh}(t) = -W_{ini}(t) \ln W_{ini}(t) - \sum_{k \neq ini}^{D} W_k(t) \ln W_k(t)$

\[
\approx -W_{ini}(t) \ln W_{ini}(t) - \left[ 1 - W_{ini}(t) \right] \ln \left[ \frac{1 - W_{ini}(t)}{N_{pc}} \right]
\]

Survival Probability

$N_{pc} = \langle \exp[S_{sh}(t)] \rangle$
Dynamic Properties: Entropies (cont.)

• Full Random Matrix:

  - Analytical and numerical data agree perfectly for random matrices
  - Linear increase for both integrable and chaotic models
  - Both entropies seem to show similar behavior
Summary and Conclusion

• Random matrices
  • Analytical results
  • References and bounds for realistic systems

• Decay of initial state:
  • Random matrices: Bessel decay (fastest)
  • Realistic Systems: Gaussian decay
    • Faster than exponential!
    • Both integrable and chaotic systems!

• Shannon Entropy
  • Linear increase: exponential spreading

• Shannon and Entanglement Entropies
  • Show very similar behaviors
  • Is there any information that can only come out of Entanglement Entropy?

• Acknowledgements
  • NSF
  • Kressel Fellowship
Back up slides
Level Number Variance

\[ \Sigma^2(l) = \frac{2}{\pi^2} \left[ \ln(2\pi l) + \gamma_e + 1 - \frac{\pi^2}{8} \right] \]

- Black points: data
- Red Curve: logarithmic
- Blue curve: linear