Realistic Many-Body Quantum Systems vs. Full Random Matrices: Static and Dynamical Properties

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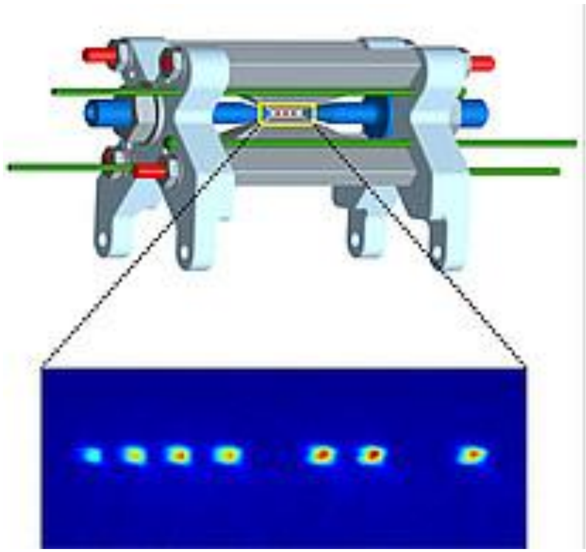
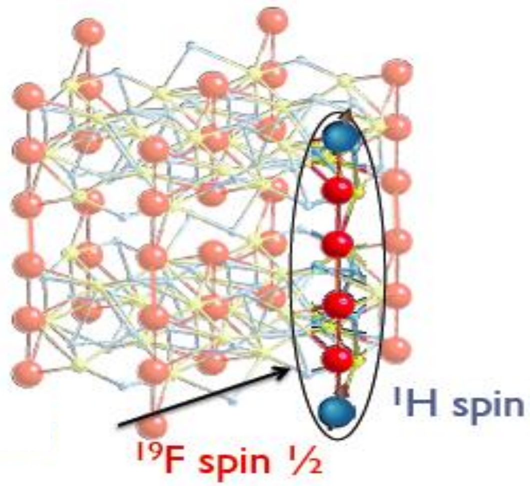
Non-equilibrium many-body quantum systems

- 1D many-body systems far from equilibrium
  - Experimentally accessible
  - Technological applications
  - Not well understood
- Goal: Understand and characterize:
  - Dynamics at different time scales
    - Dependence on chaoticity
  - Conditions for thermalization
- Use random matrices





Full Random Matrices

- Matrix filled with random numbers
  - First used by Wigner to model heavy nuclei
  - Interactions treated statistically
  - Details overlooked
  - Constrain to satisfy symmetries: real and symmetric (GOE)

- Not realistic
  - All particles interact at the same time
  - Faraway interactions as strong as nearby interactions

- Advantages
  - Analytical results
  - References and bounds for realistic systems

Realistic Hamiltonians

Integrable XXZ model	$H = H_{XXZ} + \epsilon_1 J S_1^z$
Chaotic defect model	$H = H_{XXZ} + dJS^z_{\lfloor L/2 \rfloor} + \epsilon_1 J S_1^z$
Chaotic NNN model	$H = H_{XXZ} + \lambda H_{NNN} + \epsilon_1 J S_1^z$

Parameter	Value
J	1
ϵ_1	.1
Δ	.48
d	.9
λ	1

S_1^z: small defect on first site to break symmetries

$$H_{XXZ} = J \sum_{n=1}^{L-1} (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z)$$

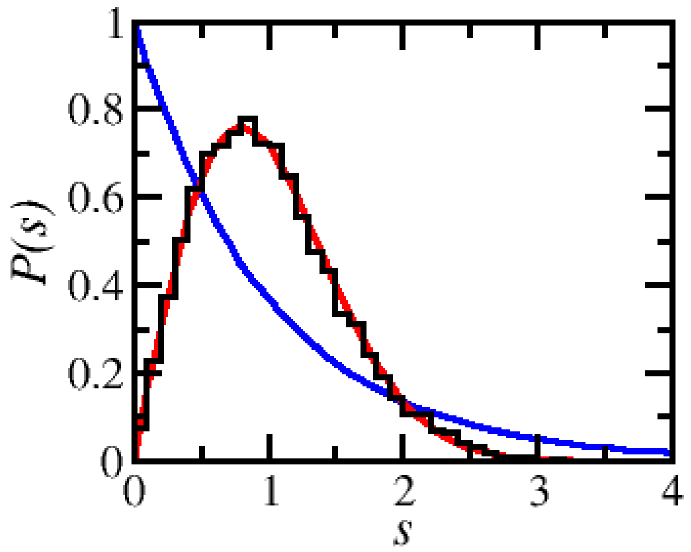
$$H_{NNN} = J \sum_{n=1}^{L-2} (S_n^x S_{n+2}^x + S_n^y S_{n+2}^y + \Delta S_n^z S_{n+2}^z)$$

$dJS^z_{\lfloor L/2 \rfloor}$: defect in the middle

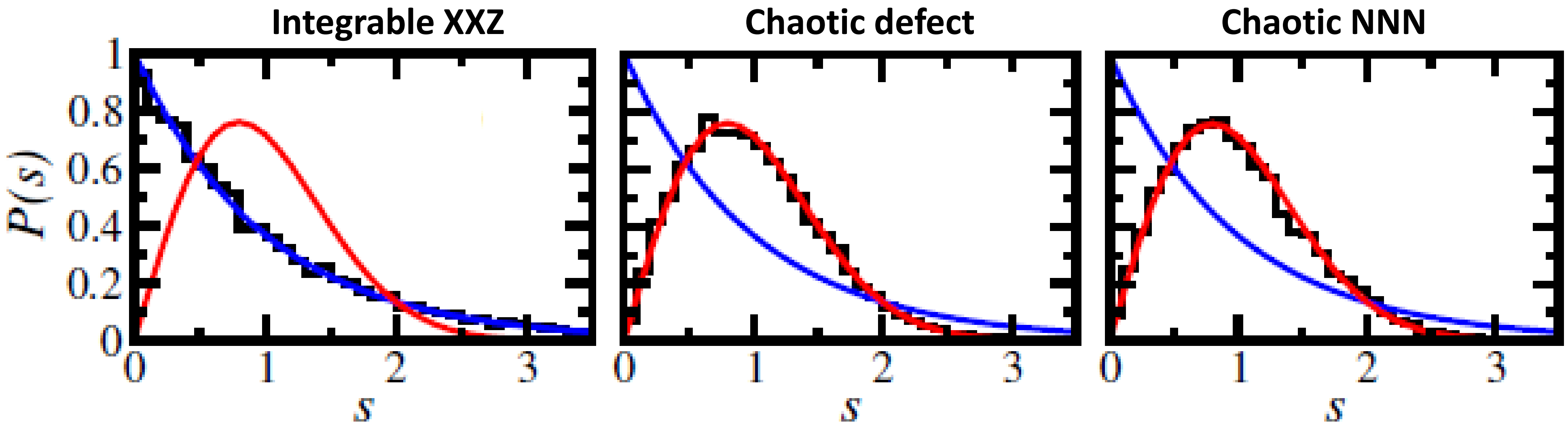
Static Properties: Level Spacing Distribution

- Full Random matrix: Wigner Dyson

$$P(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi s^2}{4}\right)$$



- Black curves: numerical data
- Red curve: Wigner Dyson
- Blue curve: Poisson

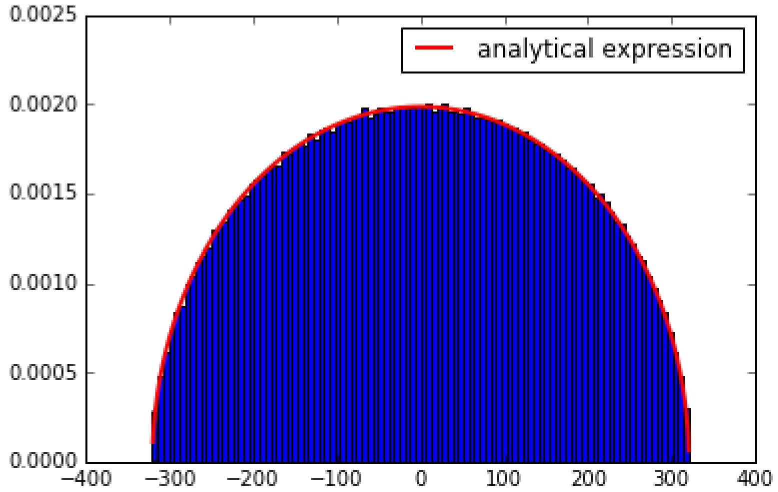


Static Properties: density of states

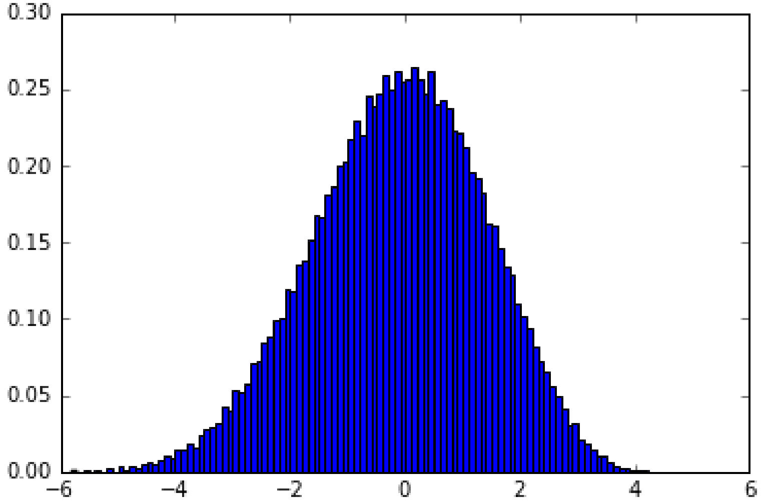
- Full Random matrix:

$$\rho^{DOS}(E) = \frac{2}{\pi\varepsilon}\sqrt{1-\left(\frac{E}{\varepsilon}\right)^2}$$

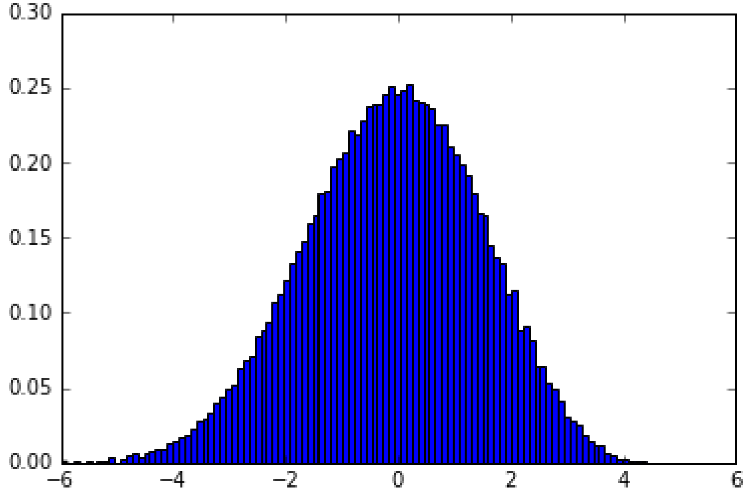
$-\varepsilon \leq E \leq \varepsilon$



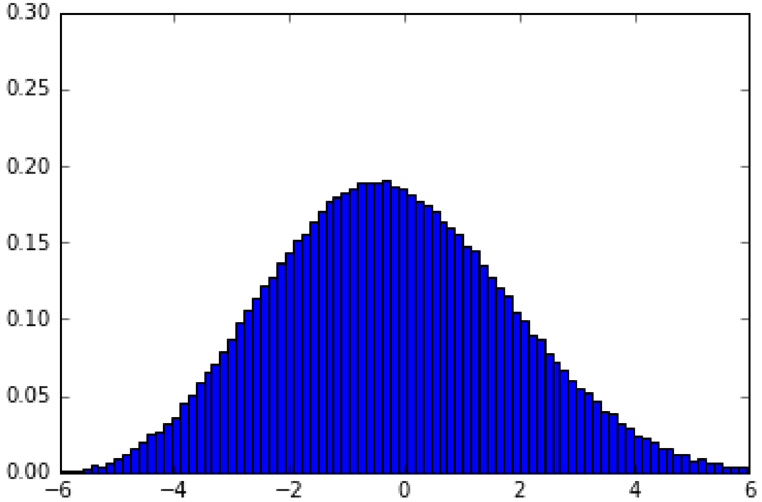
Integrable XXZ



Chaotic defect

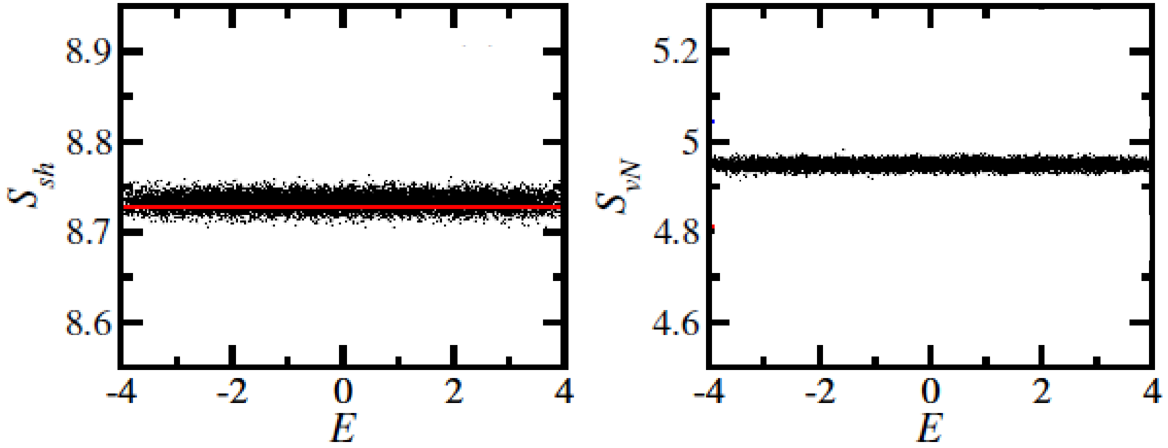


Chaotic NNN

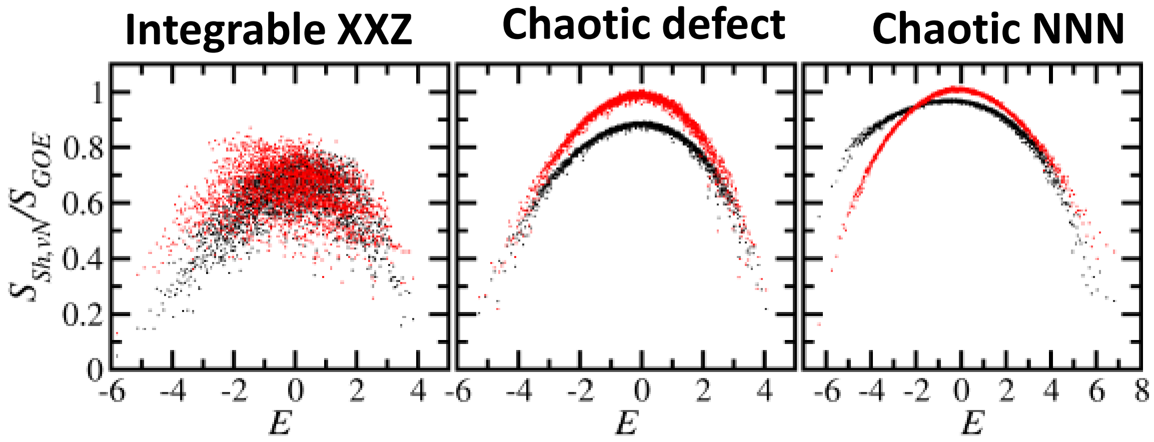


Static Properties: Entropies

- Shannon: $S_{sh}^{\alpha} = -\sum_k |C_k^{\alpha}|^2 \ln |C_k^{\alpha}|^2$
- Entanglement: $\rho_A = Tr_B(\rho)$; $\quad S_{vN} = -Tr(\rho_A \ln \rho_A)$



- Black: Numerical Data
- Red: $\ln(.48\mathcal{D})$



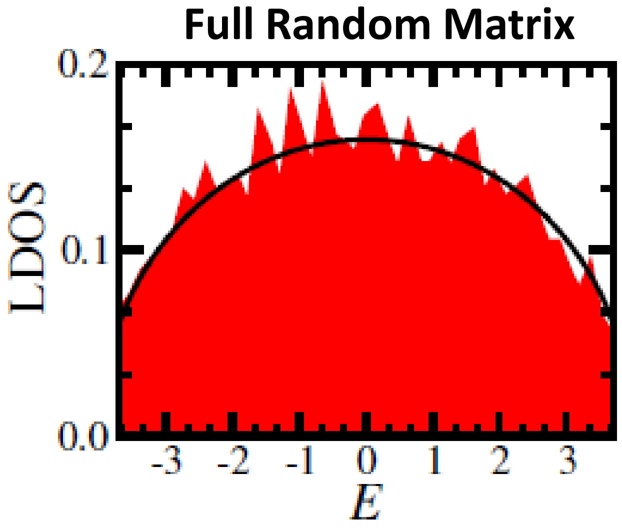
- Black: Normalized Entanglement Entropy
- Red: Normalized Shannon Entropy

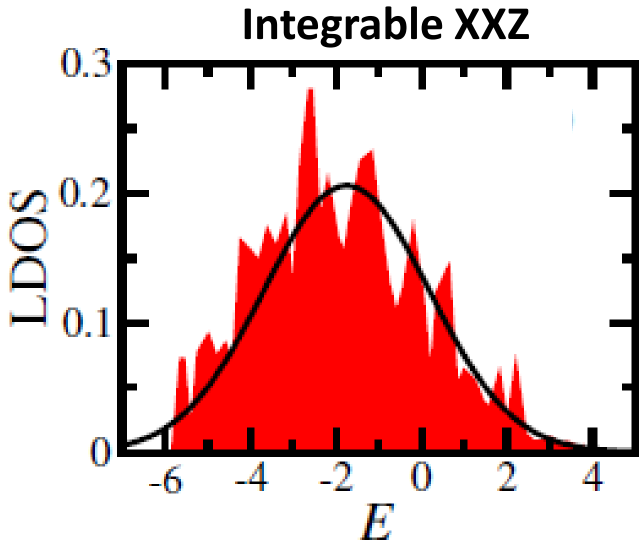
Dynamic Properties: Survival Probability

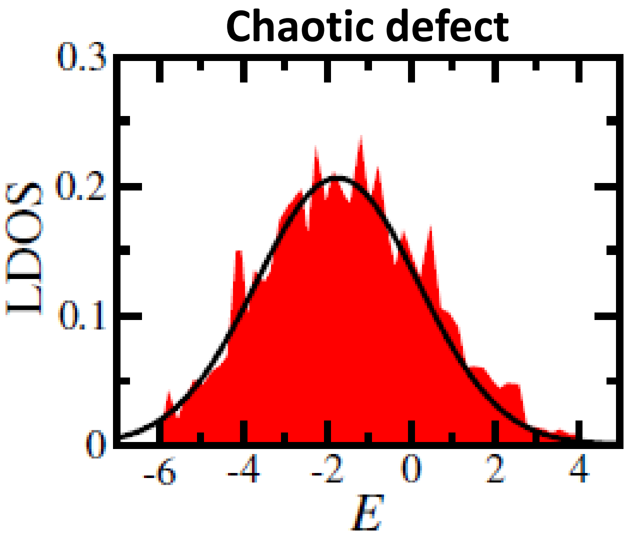
- $W_{ini}(t) \equiv \left|\langle\Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2 = \left|\int P_{ini,ini}(E)e^{-iEt}dE\right|^2$

- LDOS: $P_{ini,ini}(E) = \sum_\alpha |C_{ini}^\alpha|^2 \delta(E - E_\alpha)$

  - Initial State: Néel state: $|\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow \cdots\rangle$

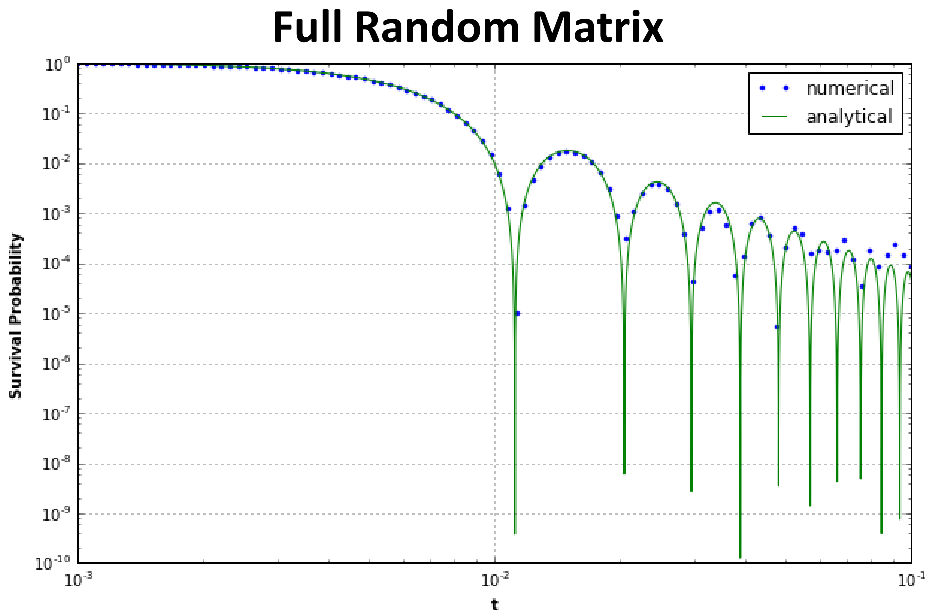




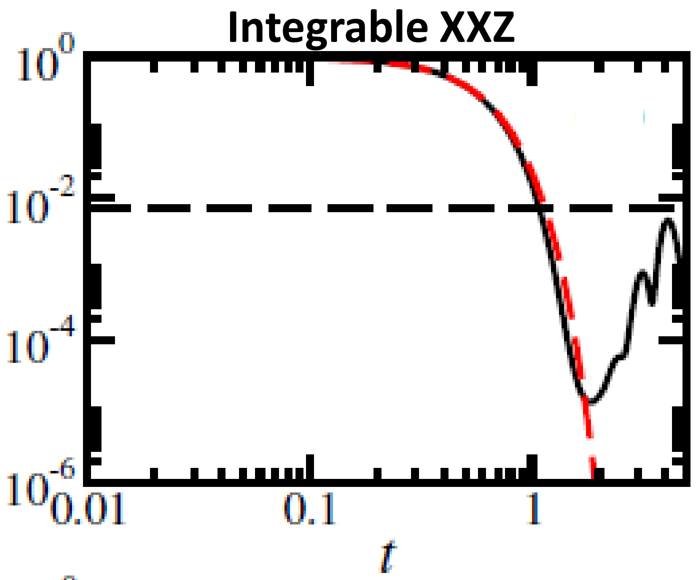


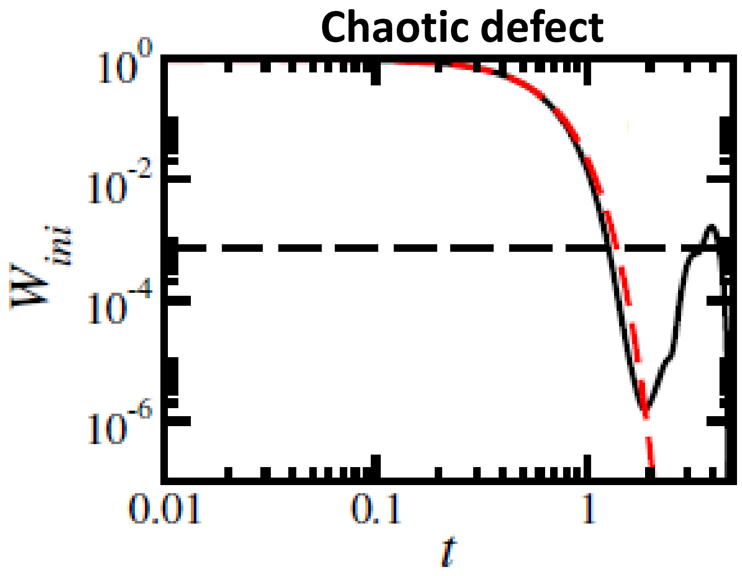
Dynamic Properties: Survival Probability

- Full Random matrix
  - $W_{int}(t) = \frac{[\mathcal{J}_1(2\sigma_{ini}t)]^2}{\sigma_{ini}^2 t^2}$
  - Bessel decay
  - Envelope $\sim t^{-3}$



- Realistic Hamiltonians:
  - Gaussian decay
    - Even integrable system
  - Black: Numerical data
  - Red: Gaussian





Dynamic Properties: Entropies

- $S_{sh}(t) = -\sum_{k=1}^{\mathcal{D}} W_k(t) \ln W_k(t)$
  - $W_k(t) = \left|\langle \phi_k \middle| e^{-iHt} \middle| \Psi(0) \rangle\right|^2$

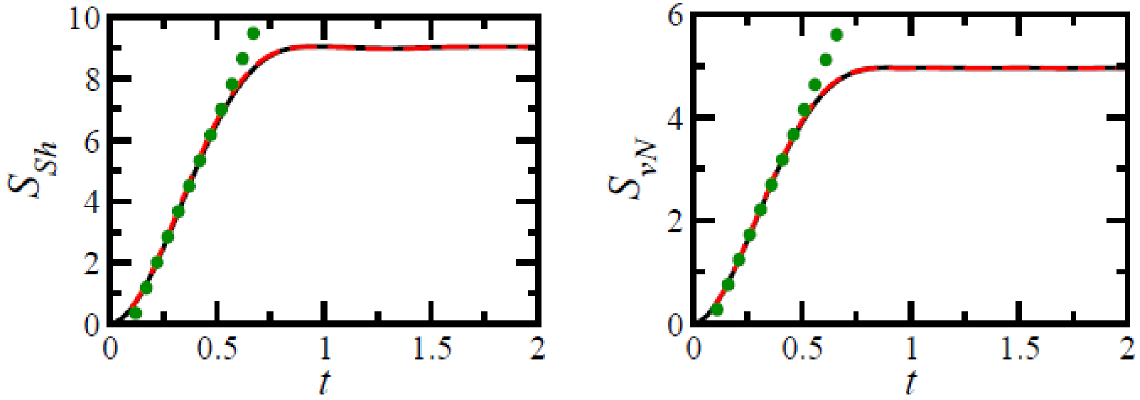
- $S_{sh}(t) = -W_{ini}(t) \ln W_{ini}(t) - \sum_{k \neq ini}^{\mathcal{D}} W_k(t) \ln W_k(t)$

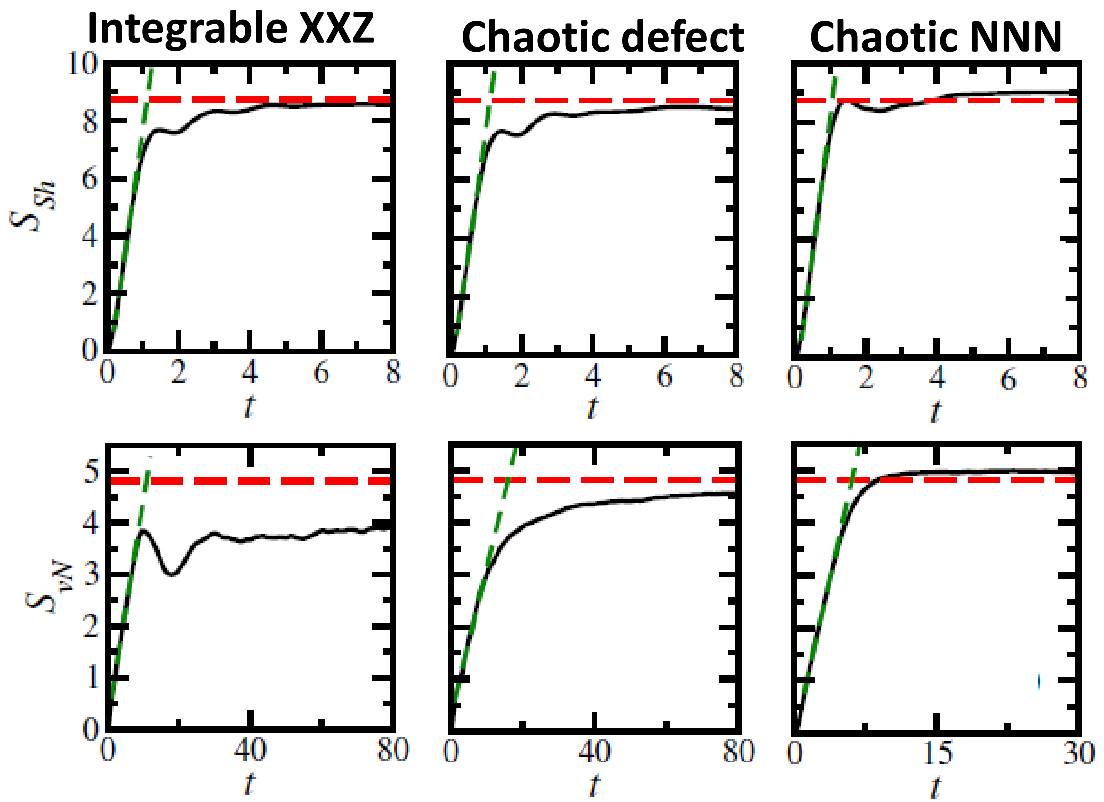
$$\approx \underbrace{-W_{ini}(t)}_{\text{Survival Probability}} \ln W_{ini}(t) - [1 - W_{ini}(t)] \ln \left[\frac{1 - W_{ini}(t)}{N_{pc}}\right]$$

$N_{pc} = \langle \exp[S_{sh}(t)] \rangle$

Dynamic Properties: Entropies (cont.)

- Full Random Matrix:





- Analytical and numerical data agree perfectly for random matrices
- Linear increase for both integrable and chaotic models
- Both entropies seem to show similar behavior

Summary and Conclusion

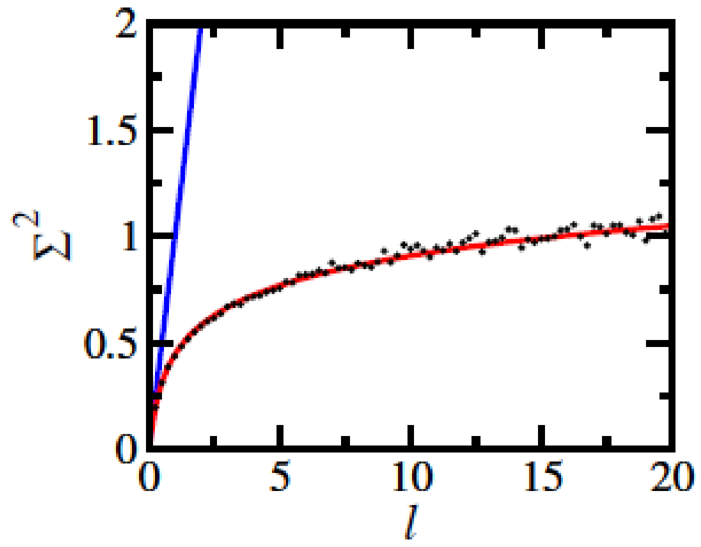
- Random matrices
  - Analytical results
  - References and bounds for realistic systems
- Decay of initial state:
  - Random matrices: Bessel decay (fastest)
  - Realistic Systems: Gaussian decay
    - Faster than exponential!
    - Both integrable and chaotic systems!
- Shannon Entropy
  - Linear increase: exponential spreading
- Shannon and Entanglement Entropies
  - Show very similar behaviors
  - Is there any information that can only come out of Entanglement Entropy?

- Acknowledgements
  - NSF
  - Kressel Fellowship

Back up slides

Level Number Variance

- $\Sigma^2(l) = \frac{2}{\pi^2}\left[\ln(2\pi l) + \gamma_e + 1 - \frac{\pi^2}{8}\right]$
- Black points: data
- Red Curve: logarithmic
- Blue curve: linear



Integrable XXZ | Chaotic defect | Chaotic NNN