Doublons Stability

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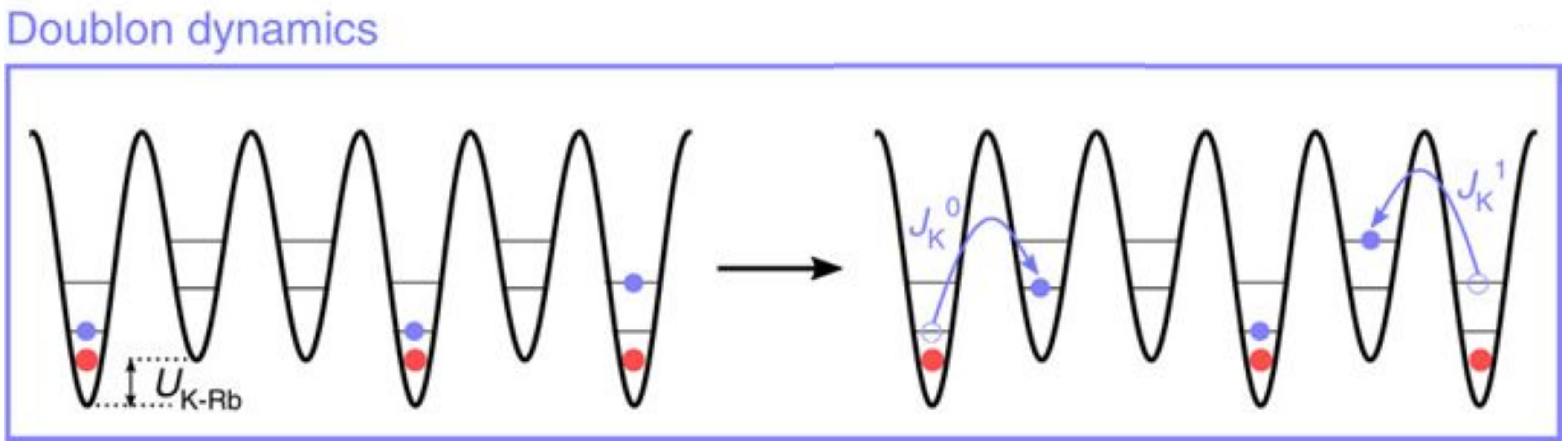
Overview

- Model chain of spins ½
- Hamiltonian, eigenstates, eigenvalues
- Dynamics
- Doublons

Doublon Experiments

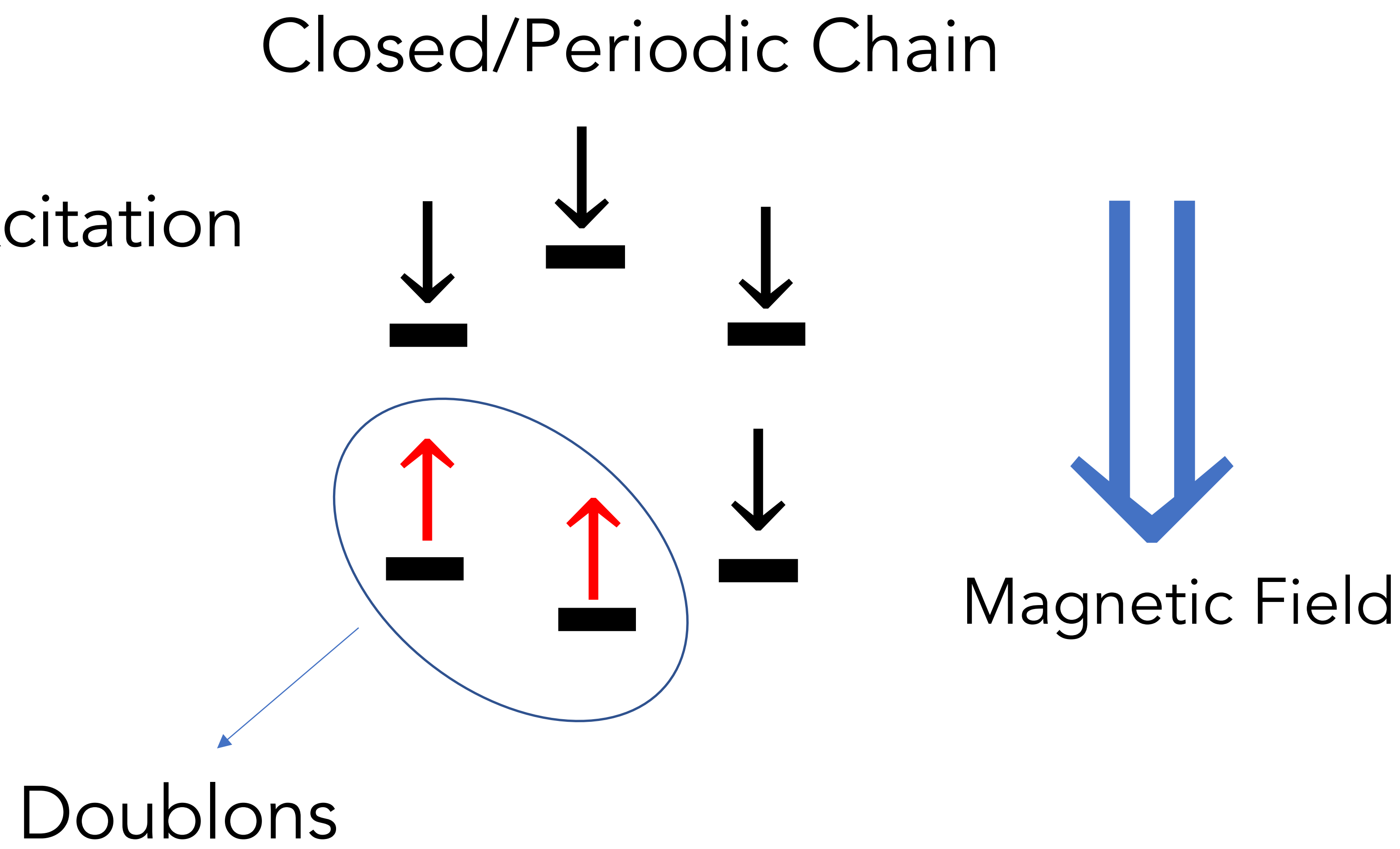


Covey et al, Nature Comm. **7**, 11279 (2016)
Winkler et al, Nature **441**, 853 (2006)

Spin ½

$\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \dashrightarrow$ Excitation

$\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



Hamiltonian of Spin ½ System

$$\hat{H} = \sum_{n=1}^{L} \frac{J}{4} \underbrace{\left[\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right]}_{\text{Flip-flop term}} + \frac{J_z}{4} \underbrace{\left[\sigma_n^z \sigma_{n+1}^z\right]}_{\text{Ising interaction}}$$

Pauli Matrices:

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Flip-Flop Term

$$\hat{H} = \sum_{n=1}^{L} \frac{J}{4}\left[\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right] + \frac{J_z}{4}\left[\sigma_n^z \sigma_{n+1}^z\right]$$

$$\sigma^x \left|\uparrow\right\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \left|\downarrow\right\rangle \qquad \sigma^y \left|\uparrow\right\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \left|\downarrow\right\rangle$$

$$\sigma^x \left|\downarrow\right\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left|\uparrow\right\rangle \qquad \sigma^y \left|\downarrow\right\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left|\uparrow\right\rangle$$

$$\frac{J}{4}\left[\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right] \left|\uparrow\downarrow\right\rangle = \frac{J}{4} \left|\downarrow\uparrow\right\rangle$$

$$\frac{J}{4}\left[\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right] \left|\downarrow\uparrow\right\rangle = \frac{J}{4} \left|\uparrow\downarrow\right\rangle$$

Ising Interaction

$$\hat{H} = \sum_{n=1}^{L} \frac{J}{4}\left[\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right] + \frac{J_z}{4}\left[\sigma_n^z \sigma_{n+1}^z\right]$$

$$\sigma^z \left|\uparrow\right\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = +\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left|\uparrow\right\rangle$$

$$\sigma^z \left|\downarrow\right\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \left|\downarrow\right\rangle$$

$$\frac{J_z}{4}\left[\sigma_n^z \sigma_{n+1}^z\right]\left|\uparrow\uparrow\right\rangle = +\frac{J_z}{4}\left|\uparrow\uparrow\right\rangle$$

$$\frac{J_z}{4}\left[\sigma_n^z \sigma_{n+1}^z\right]\left|\uparrow\downarrow\right\rangle = -\frac{J_z}{4}\left|\uparrow\downarrow\right\rangle$$

Hamiltonian In Matrix Form

basis

L=4

n=2

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \uparrow\uparrow\downarrow\downarrow & \uparrow\downarrow\uparrow\downarrow & \uparrow\downarrow\downarrow\uparrow & \downarrow\uparrow\uparrow\downarrow & \downarrow\uparrow\downarrow\uparrow & \downarrow\downarrow\uparrow\uparrow \end{array} \\
\begin{array}{c} \uparrow\uparrow\downarrow\downarrow \\ \uparrow\downarrow\uparrow\downarrow \\ \uparrow\downarrow\downarrow\uparrow \\ \downarrow\uparrow\uparrow\downarrow \\ \downarrow\uparrow\downarrow\uparrow \\ \downarrow\downarrow\uparrow\uparrow \end{array} &
\begin{pmatrix}
0 & \frac{J_{xy}}{2} & 0 & 0 & \frac{J_{xy}}{2} & 0 \\
\frac{J_{xy}}{2} & -J_z & \frac{J_{xy}}{2} & \frac{J_{xy}}{2} & 0 & \frac{J_{xy}}{2} \\
0 & \frac{J_{xy}}{2} & 0 & 0 & \frac{J_{xy}}{2} & 0 \\
0 & \frac{J_{xy}}{2} & 0 & 0 & \frac{J_{xy}}{2} & 0 \\
\frac{J_{xy}}{2} & 0 & \frac{J_{xy}}{2} & \frac{J_{xy}}{2} & -J_z & \frac{J_{xy}}{2} \\
0 & \frac{J_{xy}}{2} & 0 & 0 & \frac{J_{xy}}{2} & 0
\end{pmatrix}
\end{array}
$$

Eigenvalues and Eigenvectors

Closed Chain → L=6, n=2

J_z → Interaction strength (Ising interaction)

J_{xy} → Coupling strength (flip-flop)

Eigenstate = -50.0075

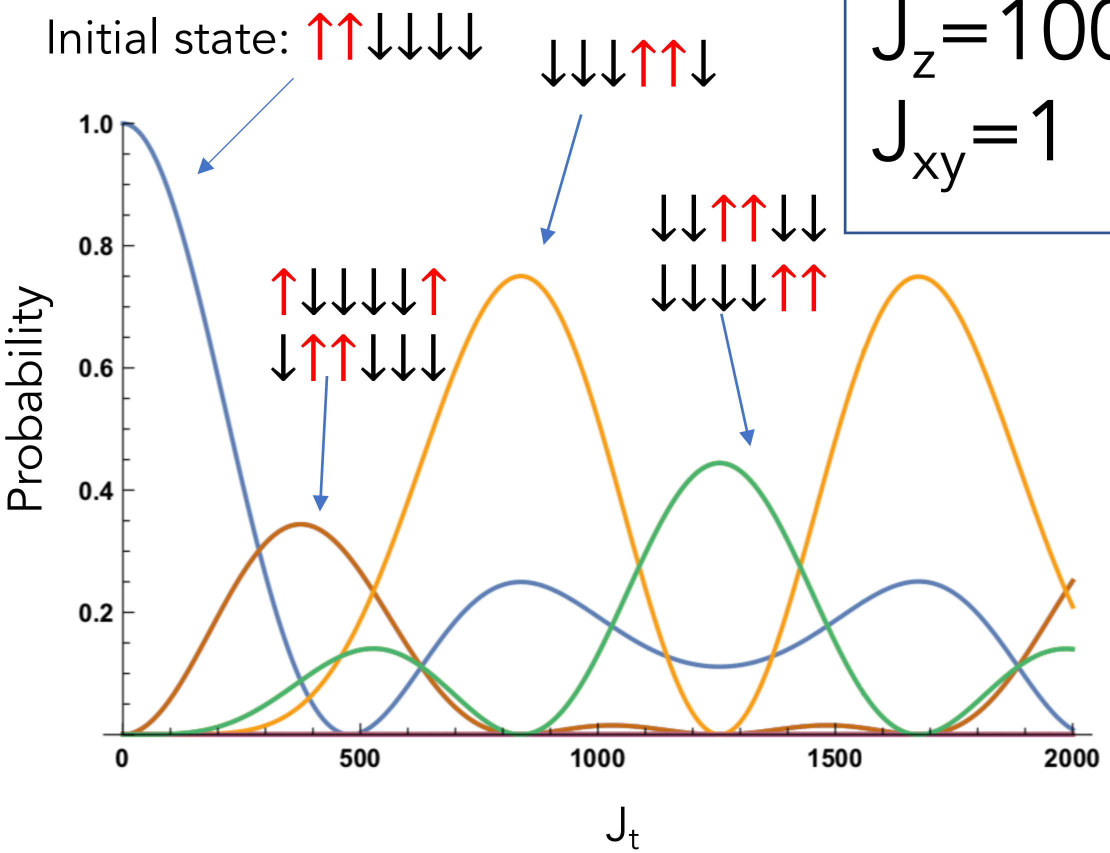
Eigenstate = 0.901388

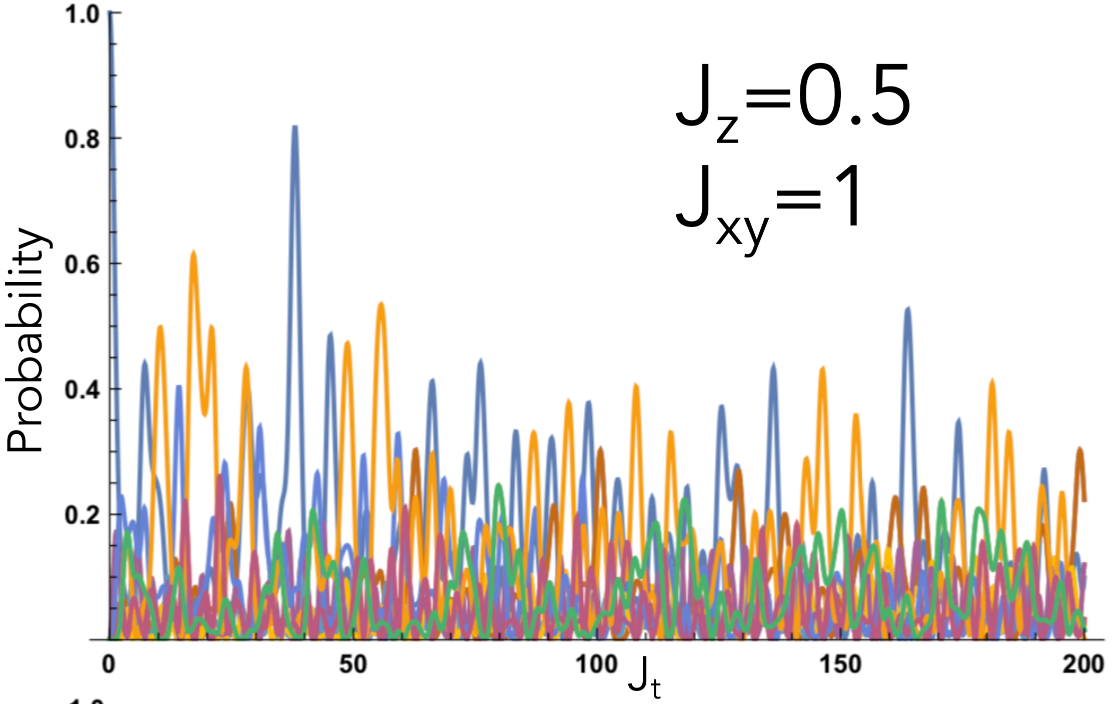
$$\boxed{\begin{array}{l} J_z=100 \\ J_{xy}=1 \end{array}} \Longrightarrow \begin{pmatrix} 0.49 \\ \sim 0 \\ -0.51 \\ \sim 0 \\ -0.49 \\ \sim 0 \\ 0.01 \end{pmatrix} \quad \begin{array}{c} \uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \\ \uparrow\downarrow\uparrow\downarrow\downarrow\downarrow \\ \downarrow\downarrow\uparrow\uparrow\downarrow\downarrow \\ \downarrow\downarrow\uparrow\downarrow\uparrow\downarrow \\ \downarrow\downarrow\downarrow\uparrow\uparrow\downarrow \\ \downarrow\downarrow\downarrow\uparrow\downarrow\uparrow \\ \downarrow\downarrow\downarrow\downarrow\uparrow\uparrow \end{array} \quad \begin{pmatrix} -0.29 \\ 0.23 \\ 0.30 \\ -0.46 \\ 0.29 \\ -0.23 \\ -0.003 \end{pmatrix} \Longleftarrow \boxed{\begin{array}{l} J_z=0.5 \\ J_{xy}=1 \end{array}}$$

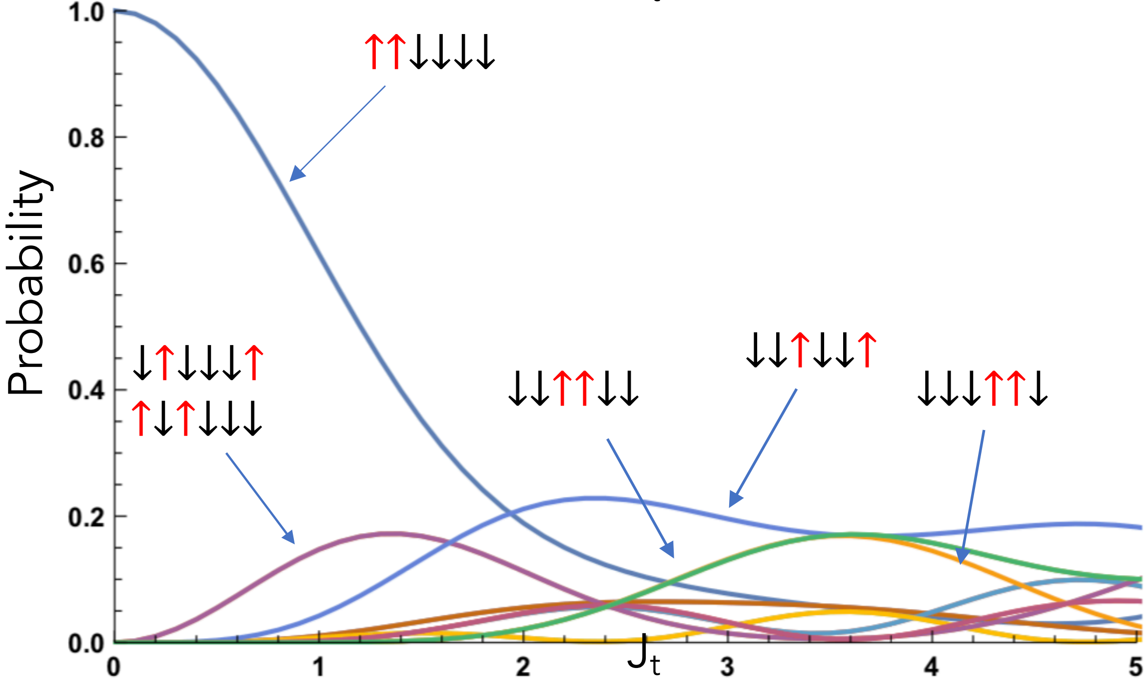
Dynamics

Closed Chain → L=6, n=2

J_z=100
J_{xy}=1







Conclusions

- Jz>>Jxy → bound pairs
- Stability of doublons
- Impurity in the chain

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