15 SECONDS TO ALPHA: CMBX RISK PRICING WITH LIMITED INFORMATION

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Abstract

This paper introduces a method for daily and intraday estimation of reduced form risk decompositions for CMBX. We estimate daily default, rates, liquidity, and excess liquidity risk partitions from simulated data in Christopoulos (2020) and Christopoulos and Jarrow (2018). We find daily liquidity and excess liquidity partitions to be significant in explaining effective bid-ask spreads from 11/2007-4/2019 and in 20-day forecasts. Within the Covid pandemic, we perform intraday CMBX risk partition estimations in intervals of 15 seconds for 240 days and reveal regular patterns of CMBX risk partition volatility in the cross-section. We fuse these insights to the related REIT sector in 54 automated long/short day trading strategies. Using the ICAPM of Merton (1990), we find 96% of our strategies exhibited statistically significant $\alpha's$, with 65% producing cumulative returns between 0.73% and 48.74%. The results of this paper support assessment of CMBX and REIT liquidity with CMBX risk partitions at daily and intraday frequencies.

Key Words: CMBX, Liquidity, Microstructure, REITs, Risk Management, Trading
JEL Codes: C58, E10, G17, R30

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Introduction

Microstructure bid-ask spread estimation is not the only theoretical measure of liquidity. The reduced form theoretical liquidity and excess liquidity partitions of commercial mortgage backed securities (CMBS), first introduced in Christopoulos (2017) and then indexed in Christopoulos and Jarrow (2018), also provide rich insights into CMBS liquidity. In this paper we reconcile classical microstructure liquidity assessments with estimates of reduced form liquidity and excess liquidity risk partitions. We extend the frequency of estimation in the previous work from monthly to daily and intraday frequencies with a new technique. In so doing, we disclose new insights into liquidity for commercial mortgage backed securities indexed credit default swaps (CMBX) and for Real Estate Investment Trusts (REITs).

In Christopoulos (2017) the reduced form simulations used in that study first allowed, and then restricted, simulation of default. This allowed for isolation of theoretical compensation for default and interest rate risks which were explicitly modelled. This extended related work in the area of Corporate bonds found in Bao, Pan and Wang (2011) and Gilchrist and Zakrajsek (2012) that identified compensation in risk premia apart from default risk.

With a set of novel transformations Christopoulos (2017) introduced the technique to project fair value prices onto observed risk premia in the market in basis points (bps) for CMBX classes. Since the model used in Christopoulos (2017) and then indexed in Christopoulos and Jarrow (2018) was well specified, the pricing of residual risk premia not associated with default and interest rate risks is interpreted theoretically as liquidity availability. Additionally, because fair value pricing is risk neutral, there are indeed instances empirically where fair value pricing so exceeds the market price, that an additional risk premia, in excess of observed risk premia, is required for reconciliation of fair value to market prices in the transformations. This additional compensation above observable risk premia is interpreted as excess liquidity availability.

Unlike default, rate and liquidity risk premia which are revealed within observable risk premia in the market, excess liquidity availability is theoretical compensation in excess of CMBX market risk premia. Excess liquidity is the amount (in bps) by which risk premia in CMBX could instantaneously increase (widen) before it experienced deterioration in regular liquidity availability. Since reduced form liquidity and excess liquidity partitions for CMBX are residual measures, they provide valuable insights into CMBX liquidity pricing apart from the idiosyncratic risks modelled at the loan and bond levels.

In the two earlier studies focusing on CMBX risk partitioning, the frequency of capture is monthly. Re-simulating and re-partitioning with the prior techniques of Christopoulos (2017) and Christopoulos and Jarrow (2018), relies on access to costly data and computation that is not currently available. As the results in those studies were quite good, we resolve in this study to increase the frequency of risk partitioning for CMBX to daily, and then to intraday pacing, in keeping with the pace of more developed markets. We do this to inquire into the nature of CMBX risks and their price formation. In so doing, we are able to leverage the earlier work and disclose new insights with our approach. We first consider, daily, the liquidity of CMBX from two perspectives: microstructure and risk partitions. We then advance the literature with the intraday risk assessments of CMBX and REITs. Both perspectives we bring of classical microstructure and estimated reduced form partitions are new to the literature concerning CMBX and REITs.

We begin by establishing daily liquidity assessments for CMBX using the classical microstructure models of Roll (1984), Thompson and Waller (1987) and variants suggested by Hasbrouck (2009), Foucault, Pagano and Roell (2013), and Christopoulos (2020). Construction of effective bid-ask spreads for CMBX establishes one perspective on CMBX liquidity. CMBX is an over-the-counter (OTC) product and is not traded on electronic exchanges. This contrasts with Corporate bonds, which are increasingly traded electronically and which are required to post execution prices and volumes on the Trade Reporting and Compliance Engine (TRACE). As such, the data over the sample period 11/2007-4/2019 we use is restricted to end of day mid-market CMBX spreads. Since these classes of microstructure models provide insights into liquidity for instruments limited to end of day mark to market values, they are well suited to CMBX liquidity assessment, from a microstructure perspective.

Next, we develop the alternative perspective on CMBX liquidity by estimating daily the reduced form liquidity and excess liquidity partitions. We use these risk partitions of liquidity to compare to the microstructure perspective on liquidity articulated in effective bid-ask spreads. We construct a linear estimation model (the 'Daily Model') using as training data the monthly simulated values of risk partitions from Christopoulos and Jarrow (2018) and the monthly market variables of the VIX, interest rates, and REIT prices. With two perspectives on CMBX liquidity established, we then test their relationships to validate the Daily Model. Our testing of the Daily Model gives us our first two main results.

First, liquidity risk partitions play a significant role in CMBX price formation across AAA and BBB credits over 2828 daily observations from 11/2007 - 4/2019. We establish this result through the use of multivariate OLS of risk premia changes regressed onto changes in the macroeconomy for observed market risk premia over 5-day rolling estimation periods to assess whether estimates of reduced form liquidity and excess liquidity measures are being communicated in effective bid-ask spreads.

Second, estimated reduced form liquidity measures have a significant effect on future observed CMBX liquidity (as measured by effective bid-ask spreads) over 20 day forecast periods. We establish this result in time-series analyses using vector autoregression (VAR) and Granger causality techniques. The significant results in those tests motivate the use of impulse response functions (IRF) to project effective bid-ask spreads in response to shocks of liquidity and excess liquidity.

In conjunction, the first two main results and supporting evidence validate the Daily Model and provide new insights into the nature of liquidity for CMBX from both microstructure and risk partition perspectives. This allows us to move forward with the Intraday Model for risk partition estimation across 240 trading days during the Covid-19 pandemic period between 4/2020-4/2021. The Intraday Model increases the speed of risk partitioning assessment by increasing the frequency of polling of market data to 15 second intervals intraday for the explanatory variables (VIX, rates, and REITs) and using their changes to estimate risk partitions, intraday.

Since CMBX and CMBS do not trade electronically, the frequency of CMBX risk partition generation in this study exceeds the market's pace of trading in CMBS and CMBX. That is interesting because it means that our intraday proxies of CMBX risk neutral risk partitions have no corresponding intraday CMBX market price for comparison. In this sense, the intraday risk partitions are empirical renderings of purely theoretical price changes. This observation invites testing of the Intraday Model, which gives rise to our third main result.

Third, during the pandemic, we find considerably greater volatility in risk partition pricing at the start of each trading day for all risk partitions compared to the end of each trading day. We establish this result by conducting a cross-sectional evaluation of millions of cumulative intraday changes in CMBX risk partitions in 15 second intervals for 240 trading days. The phenomena observed in our third main result invites further testing. We note that REITs, like CMBX, are also exposed to commercial real estate risks. Additionally, REIT trading frequency more closely matches the frequency of estimation of the risk partitions. With these insights we are able to establish our fourth main result.

Fourth, well constructed intraday trading strategies using CMBX risk partitions applied to the REIT market can result in substantial extraordinary returns and bring insights into the relative pricing of liquidity in both CMBX and REIT sectors. We establish our fourth main result by fusing the cross-sectional insights into CMBX risk partition volatility with the more frequently traded REIT sector in a set of 54 long/short day trading strategies during the Covid pandemic for each of the investment grade credit rating classes (AAA, AJ/AS, AA, A, BBB and BBB-). The strategies exploit regular and systemic aberrations in volatility in both CMBX risk partitions and REIT pricing. Using use the Intertemporal Capital Asset Pricing Model (ICAPM) of Merton (1990) we find 96% of the strategies produced positive and statistically significant $\alpha's$. 65% of those strategies also exhibited positive cumulative returns over the Covid-19 crisis period ranging from 0.73% to 48.74% with generally solid Sharpe ratios. In conjunction, the third and fourth results and their supporting evidence validate the Intraday Model and, like the Daily Model, also provide new insights into the nature of liquidity for CMBX.

The size of the CMBX and CMBS sectors stands at approximately $700 billion while the US REIT sector stands at approximately $1.3 trillion. Our paper establishes that CMBX risk partitions at daily and intraday frequencies provide new insights into the liquidity for CMBX and REIT sectors which have a combined value of about $2 trillion. As such, we provide a new perspective on the pricing of liquidity risks for two large and related sectors within real estate capital markets. The remainder of this paper is organized as follows: Section 1 discusses the data used in this study. Section 2 provides a literature review of a few existing models used to assess CMBX liquidity. Section 3 introduces the Daily Model and discusses the various statistical methods and results used for its validation. Section 4 introduces the Intraday Model applied to the Covid-19 period and provides the cross-sectional visualizations, trading strategies, and ICAPM results used for its validation. Section 5 summarizes with suggestions for future work. The Online Appendix provides supplementary information.

1 Data

In this section we discuss the data used throughout this study. The study uses data from three time frames: the training period, the daily period, and the intraday period.

1. The training period consists of 92 monthly observations from 11/2007-6/2015 in which training data is used to train the Daily Model in Section 3.
2. The daily period consists of 2828 daily observation in which data are used to produce risk partition estimates with the Daily Model in Section 3 from 11/2007-4/2019 and are used in OLS, VAR, Granger and IRF analyses.
3. The intraday period consists of 246 dates during the Covid-19 pandemic in which time stamped intraday data are used to produce estimates of risk partitions with the Intraday Model in Section 4 from 4/2020-4/2021 which are used in cross-sectional analyses and ICAPM trading tests at the open and close of the trading days.

Tables and figures summarizing the data and results are introduced in the relevant sections where the data are used.

1.1 Private data - CMBX risk premia

The private data was generously donated to us by one of the largest asset managers for our research.

Like CMBS, CMBX trade on a 'Spread' basis, where Spreads are mid-market risk premia S_t, in excess of US Treasury swap spreads. These Spreads are contributed by dealers to Markit at the close of each trading day. For all daily observation dates, t, the risk premium, S_t, captures the drivers of uncertainty on CMBX derivatives in excess of the relevant risk-free rate. The underlying collateral tranches securing CMBX used in this study represent approximately \$400 billion (bn) of the ~\$600 bn CMBS market. As a securitized credit default swap, as described in Driessen and Van Hemert (2012) and Christopoulos and Jarrow (2018), series of CMBX classes (or tranches) capture cashflows of representative assets of 25 CMBS tranches for corresponding credit ratings. A Bloomberg post and primer discussion of CMBX bid-ask spreads is provided in Appendix A.1. To our knowledge, such indications of CMBX liquidity are only available as sporadic posts by dealers to clients and are not regularly recorded and verified with time stamps intraday.

1.2 Public market data

1.2.1 Monthly

We use prices for 25 representative publicly traded REITs obtained from Yahoo! Finance that we use for the estimation of the Daily Model in Section 3 and the estimation of the Intraday Model in Section 4.

Table 1: Sample REITs

PropType	Ticker	Factor Name	REIT Name	Market Cap ($bn)
Industrial (IN)	DRE	IN_DRE	Duke Realty	$18.04
	FR	IN_FR	First Industrial Realty Trust	$6.90
	PLD	IN_PLD	Prologis, Inc.	$90.25
	SELF	IN_SELF	Global Self Storage, Inc.	$54.28
Hotel (LO)	HST	LO_HST	Host Hotels & Resorts, Inc.	$12.41
	MAR	LO_MAR	Marriott International, Inc.	$45.64
	WH	LO_WYND	Wyndam Hotels & Resorts, Inc.	$6.82
	MGM	LO_MGM	MGM Resorts International	$21.53
Multifamily (MF)	AVB	MF_AVB	Avalon Bay Communities, Inc.	$29.87
	ELS	MF_ELS	Equity LifeStyle Properties, Inc.	$13.81
	EQR	MF_EQR	Equity Residential	$29.39
	UDR	MF_UDR	UDR, Inc.	$14.80
Office (OF)	BXP	OF_BXP	Boston Properties, Inc.	$18.70
	CLI	OF_CLI	Mack-Cali Realty Corporation	$1.55
	HIW	OF_HIW	Highwoods Properties, Inc.	$4.85
	SLG	OF_SLG	SL Green Realty Corp.	$5.72
	VNO	OF_VNO	Vornado Realty Trust	$9.24
Mixed Use/Other (OT)	BKD	OT_BKD	Brookdale Senior Living Inc.	$1.54
	NNN	OT_NNN	National Retail Properties, Inc.	$8.41
	PSB	OT_PSB	PS Business Parks, Inc.	$4.15
	WPC	OT_WPC	W.P. Carey Inc.	$13.93
Retail (RT)	KIM	RT_KIM	KIMCO Realty Corporation	$9.14
	REG	RT_REG	Regency Centers Corporation	$11.05
	SPG	RT_SPG	Simon Property Group	$43.06
	TCO	RT_TCO	Taubman Centers Inc.	$3.40
			Total Sample Market Cap	$478.45
			Total Sample number	25

This table summarizes the 25 REITs used in this study. The market capitalization of the REITs are $478.45 billion as of 6/27/2021. The first column provides the property type and groups the REITs by property type separated by borders. The six property types are Industrial (IN), Hotel (LO), Multifamily (MF), Office (OF), Mixed Use/Other (OT), and Retail (RT). The second column provides the stock market ticker symbol. The third column provides the factor name composite of the property type with the ticker. The fourth column provides the name of the REIT, while the fifth column the market capitalization.

We select these REITs due to their similarity to CMBX on a few dimensions. Table 1 lists the names of the 25 REITs used in this study along with their property type, ticker, factor name (proptype_ticker), and market capitalization as of 6/27/2021. The market capitalization of these 25 REITs selected total $478.45 billion. According to the National Association of Real Estate Investment Trusts (NAREIT), the total market capitalization

Table 2: Liquidity summary for several sectors

Sector	ADV ($Bn)	Outstanding ($Bn)	Turnover	Turnover % Tsy
US Tsys	$565.00	$19,300.00	2.9275%	100.0000%
US RMBS (Agency)	$289.80	$9,900.00	2.9273%	99.9936%
US REITs	$8.70	$1,300.00	0.6692%	22.8604%
US Corporates	$27.46	$10,600.00	0.2590%	8.8480%
US CMBS	$0.84	$596.40	0.1408%	4.8112%

This table provides summary issuance and trading volume for US Treasuries, US Agency backed mortgage backed securities, US REITs, US Corporate Bonds and US CMBS. The columns report the average daily trading volume (ADV) at of Q1 2021 and the outstanding issuance, in $billions. The Turnover is the ratio of ADV/Outstanding issuance. The Turnover % of Tsy is the ratio of Turnover/Turnover of US Treasuries. Source: SIFMA and NAREIT

(market cap) of US REITs was $1249.19 billion as of December 31, 2020[1], and so our sample captures about 38% of the US REIT sector by market cap. NAREIT also states that the total number of US REITs is 223, and so by counts, our sample represents about 11% of the US REIT sector. Also, as the CMBX in our sample represent approximatly $400 billion of CMBS underlying, the size of our REIT sample and our CMBX sample are of similar market magnitude. While of similar sizes, their liquidities appear to differ. At a higher level, combining data from Securities Industry and Financial Markets Association (SIFMA) and NAREIT we see that REITs in the aggregate appear to be more liquid than CMBS. Table 2 indicates approximately 10x the dollar volume for REITs compared with CMBS with 4x the efficiency of turnover rates defined as average daily volume (ADV) divided outstanding issuance. Finally we select our REITs from multiple property types and a further summary of the sample of REITs used in our study broken down by property type is provided in Table 3.

Table 3: Aggregate Summary of Sample REITs

Property Type	Market Cap ($bn)	% Market Cap	Count	US Count	Sample % of US
Industrial (IN)	$169.47	35.42%	4	13	30.77%
Hotel (LO)	$86.39	18.06%	4	13	30.77%
Multifamily (MF)	$87.86	18.36%	4	20	20.00%
Office (OF)	$40.06	8.37%	5	19	26.32%
Mixed Use/Other (OT)	$28.02	5.86%	4	126	3.17%
Retail (RT)	$66.65	13.93%	4	32	12.50%
Total	$478.45	100.00%	25	223	100.00%

This table aggregates values related to the 25 REITs used in this study. The first column gives the property type and total labels. The six property types are Industrial (IN), Hotel (LO), Multifamily (MF), Office (OF), Mixed Use/Other (OT), and Retail (RT). The second column provides the market capitalization (Market Cap). The third column provides proportion of each property type Market Cap compared to the total Market Cap in the sample. The fourth column provides the counts of the REITs by property type in the sample. The fifth column the number of REITs by property type in the US. The sixth column captures the proportion of the sample count to the US count of REITs.

[1] See https://www.reit.com/data-research/reit-market-data/us-reit-industry-equity-market-cap

While no sample selection is perfect, we feel our selection is broadly representative of the US REIT sector. The 25 REITs are included in the training data and then used in the Daily and Intraday Models to estimate CMBX risk partitions. Table 6 in Section 3 provides summaries for the 92 dates used as the monthly training data for the Daily Model. These include monthly historical pricing for the 25 REITs as well as the Chicago Board of Option Exchange volatility index (VIX), US treasury yields, and the simulated CMBX risk partitions in spread form Christopoulos and Jarrow (2018).

1.2.2 Daily

For the period 11/2007-4/2019, we also have public daily time series from the Federal Reserve Bank of St. Louis FRED system. These data include benchmark corporate bond credits spreads for AAA and BBB ratings, the VIX volatility index, constant maturity (CMT) US Treasury yields to maturity. Following the estimation of the Daily Model, we also provide summary statistics for the public, private and estimated data of CMBX bid-ask spreads and CMBX risk partitions in Table 12. Finally, for the trading strategies and related ICAPM testing conducted during the Covid-19 pandemic period (246 days between 4/2020-4/2021) we obtain daily values of the market portfolio (Mkt-Rf, which consists of all NYSE, AMEX, and NASDAQ firms), small minus big (SMB), high minus low (HML), and momentum (MOM) indices from Ken French's website.[2]

1.2.3 Intraday

During the Covid-19 pandemic period of 4/2020-4/2021 captured the daily values of the VIX, US Treasuries, and REITs during this period using the RapidAPI application with Yahoo! Finance. These data are summarized in Tables 17 and 18 at the start and at the end of the trading days during this period. We poll the data in 15 second intervals and the average delay over this period is about 1.15 seconds. The data is always 'most-recent' and reflective of updates driven by trading in the marketplace. The data polling for each day starts at approximately 9:30:15am EST and ends at approximately 4:15:00pm EST. There were some delays in reporting due to electronic communication lags on the internet in the application of up to 2 seconds. Additionally, some delays in reporting can take of up to 15

[2]See https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

minutes for the VIX at the start of each trading day. All Federal holidays and are excluded with trading only taking place on those days where both the bond market and stock market are open. Reporting terminates early with early market closings associated with holidays or stock market circuit breaker triggers.

2 Literature review

This section discusses some of the relevant literature in the domain of pricing liquidity.

2.1 Bid-ask measures of liquidity

Bid-ask spreads represent the difference between the quotes of where market-makers stand ready to buy (bid-side) and sell (ask-side aka 'offered-side') securities at a specific point in time. While bid-ask spreads provide a rich measure of a security's liquidity, it stands to reason, that infrequently traded securities (where bid-ask spreads are less frequently observable) may present investors with greater opacity with respect to trading information which may, in turn, compound their illiquidity as noted in Hasbrouck (2009) and Fong, Holden and Trzcinka (2017).

Many of the classical microstructure models that investigate into issues of illiquidity are based on price and volume information content, as introduced in Kyle (1985), Easley, et al (1996), and O'Hara (1997) and more recently Abdi and Ranaldo (2017). As a result, most studies in microstructure focus on products such as publicly traded equities where price and volume data is available for empirical evaluation. However, this rich data is simply unavailable for many credit sensitive fixed income instruments, particularly in the securitized sector. Despite their size relative to equities, US fixed income markets can suffer from considerable illiquidity, particularly during times of stress as noted in Bao, Pan and Wang (2011) and Bao, O'Hara and Zhou (2018). This may be due to the over-the-counter (OTC) trading mechanism as discussed in Campello, Chen and Zhang (2008) and the comparatively less frequently traded characteristic for much of credit sensitive fixed income compared with equities as noted in Goldstein, Jiang and Ng (2017).

One sector with such limited pricing information is the CMBS sector and its indexed derivatives, CMBX. CMBX is an OTC indexed credit default swap which does not trade on electronic exchanges. Communication between market makers (dealers) and investors is done principally through Bloomberg terminal posts and sporadically.[3] Some data improvement in the area of Corporates has helped to advance the fixed income microstructure literature [4] and there have also been some advances in data and analysis for securitizations as discussed in Hollified, Nekyudov and Spatt (2017). Nevertheless, many credit sensitive fixed income instruments, such as CMBX, still provide only end of day quotes with only sporadic intra-day updates and very limited recording and data sharing. As a result, the technology to estimate bid-ask spreads for CMBX from a microstructure perspective is restricted to Roll (1984), Harris (2003), Hasbrouck (2009), Thompson and Waller (1987) and, most recently, Christopoulos (2020).

Since microstructure liquidity estimates provide insights into a product's liquidity we implement four models for CMBX bid-ask spreads in this study which we will use to compare to estimates of reduced form theoretical liquidity to validate our daily estimation model of risk partitions. The microstructure models we use are Roll (1984), a restricted adaptation of Roll (1984) suggested by Hasbrouck (2009), the moving average approach of Thompson and Waller (1987), and the new adaptation of Roll (1984) introduced in Christopoulos (2020) derived using complex numbers defined as the *Absolute Roll Measure*, $\hat{s} = 2\sqrt{|-\mathrm{COV}\left(\Delta P_t, \Delta P_{t+1}\right)|}$.

The seminal work on bid-ask spreads derived from closing end-of-day prices is found in Roll (1984). As discussed in Lo and Wang (2000), solving for the effective bid-ask spread, s

$$\begin{aligned}
-\frac{s^2}{4} =& \mathrm{COV}\left(\Delta P_t, \Delta P_{t+1}\right) \\
s^2 =& -4\mathrm{COV}(\Delta P_t, \Delta P_{t+1}) \\
\sqrt{s^2} =& \sqrt{-4\mathrm{COV}(\Delta P_t, \Delta P_{t+1})}
\end{aligned}$$

$$s = 2\sqrt{-\mathrm{COV}(\Delta P_t, \Delta P_{t+1})} \tag{1}$$

yields a complex number when the first order autocovariance $\mathrm{COV}\left(\Delta P_t, \Delta P_{t+1}\right) > 0$.

Roll (1984) and Harris (1990) address this problem by treating the value of s differently in different domains. Following Harris (1990), values for s^+ apply to instances of positive

[3] See Appendix A.1.

[4] See Hotchkiss and Ronen (2002), Bao, Pan and Wang (2011), Han and Zhou (2016), and Haddad, Moreira and Muir (2020) among others.

autocovariance, while values for s^- apply to instances of negative autocovariance, such that:

$$s = \begin{cases} s^+ = -2\sqrt{\text{COV}(\Delta P_t, \Delta P_{t+1})} & \text{for COV}(\Delta P_t, \Delta P_{t+1}) > 0 \\ s^- = 2\sqrt{-\text{COV}(\Delta P_t, \Delta P_{t+1})} & \text{for COV}(\Delta P_t, \Delta P_{t+1}) \leq 0 \end{cases} \tag{2}$$

This has the effect, as noted in Lo and Wang (2000) of 'preserving the sign of the covariance' in keeping with the empirical analyses of Roll (1984) and Harris (1990). However this preservation of sign also results in negative effective bid-ask spread estimates, s, as shown in Eq. (2). Negative bid-ask spreads imply market-makers inverting markets; standing ready to buy securities at *higher* prices than where they would sell them which would be ruinous to market makers. Additionally, since empirical observations of positive autocovariance in prices are frequently observed between 28% and 50% of observations in prior studies, reliance on Roll (1984) faces well known limitations.

Christopoulos (2020) addresses these limitations differently through the development of the Absolute Roll Measure.[5] Our derivation below is more succinct. Consider the identities for the complex number i are given[6] by

$$\sqrt{x} = i\sqrt{-x}\; \forall\, x \in \mathbb{R} \tag{3}$$

and

$$|ix| = |x|\; \forall\, x \in \mathbb{R} \tag{4}$$

Since the effective bid-ask spread, s, in Eq. (1) is undefined on the set of real numbers whenever $t \in \mathbb{R} : \text{COV}(\Delta P_t, \Delta P_{t+1}) > 0$, we resolve with Eqs. (3) and (4). Motivated by Harris (1990), we partition the domain of Eq. (1) into instances of positive and negative autocovariance, s^+ and s^-, respectively. This allows us to propose a new alternative measure to estimate the bid-ask spread, the *Absolute Roll Measure*, $\hat{s}$, defined as

[5] A full derivation of the Absolute Roll Measure from Christopoulos (2020) is found in Appendix A.2.
[6] See Simon and Blume (1994) for example.

$$\hat{s} = |s| = \begin{cases} s^{+} = \left|2i\sqrt{\mathrm{COV}(\Delta P_t, \Delta P_{t+1})}\right| & t \in \mathbb{R}\text{:}\mathrm{COV}(\Delta P_t, \Delta P_{t+1}) \geq 0 \\ s^{-} = \left|2\sqrt{-\mathrm{COV}(\Delta P_t, \Delta P_{t+1})}\right| & t \in \mathbb{R}\text{:}\mathrm{COV}(\Delta P_t, \Delta P_{t+1}) < 0 \end{cases}$$

$$= \begin{cases} s^{+} = 2\sqrt{\mathrm{COV}(\Delta P_t, \Delta P_{t+1})} & t \in \mathbb{R}\text{:}\mathrm{COV}(\Delta P_t, \Delta P_{t+1}) \geq 0 \\ s^{-} = 2\sqrt{\mathrm{COV}(\Delta P_t, \Delta P_{t+1})} & t \in \mathbb{R}\text{:}\mathrm{COV}(\Delta P_t, \Delta P_{t+1}) < 0 \end{cases} \tag{5}$$

$$\text{Since } s^{+} = s^{-} \;\therefore\; |s| = 2\sqrt{|\mathrm{COV}(\Delta P_t, \Delta P_{t+1})|}\;\forall\,\mathbb{R}, \forall\, t$$

$$\text{But} \quad 2\sqrt{|\mathrm{COV}\,(\Delta P_t, \Delta P_{t+1})|} = 2\sqrt{|-\mathrm{COV}\,(\Delta P_t, \Delta P_{t+1})|}\;\forall\,\mathbb{R},\forall\, t$$

$$\therefore \hat{s} = 2\sqrt{|-\mathrm{COV}\,(\Delta P_t, \Delta P_{t+1})|}\;\forall\,\mathbb{R}, \forall\, t \;\blacksquare$$

This Absolute Roll Measure, $\hat{s}$, guarantees a strictly non-negative bid-ask spread observable for all price changes in all traded asset markets. For observations where $\mathrm{COV}(\Delta P_t, \Delta P_{t+1}) \geq 0$, the interpretation is the corresponding effective bid-ask spread is the magnitude of s, regardless of whether s is real or imaginary, as found in Christopoulos (2020).

Finally, an alternative bid-ask spread estimator model we consider is the model of Thompson and Waller (1987). This model also uses end-of-day prices to estimate bid-ask spreads from the absolute value of 5-day moving averages of changes in end-of-day closing prices, ($\overline{|\Delta p_t|} = \frac{1}{5}\sum_{t=1}^{5} |\Delta p_t|$). The model is practical in that it guarantees a strictly non-negative bid-ask estimator result. It is also actively utilized in policy and by practitioners, as discussed in He and Mizrach (2017).

2.2 Reduced form measures of liquidity

While bid-ask spreads represent one measure of securities liquidity, there are others. For example, Amihud (2002) and Gilchrist and Zakrajsek (2012) each identify measures of liquidity for credit sensitive fixed income securities. In particular, certain areas of the literature are focused on disclosing liquidity drivers. Bao, Pan and Wang (2011) for example isolate factors apart from credit risk to be attributable to overall risk premia for that sector. They find illiquidity in Corporate bonds to increase with volatility and to be highly time varying. Gilchrist and Zakrajsek (2012) reveal statistically the excess bond premia in excess of default risk estimated using firm specific variables and the distance to default framework Merton

(1974). Their results indicate significant explanatory power for both the isolated default partition and the excess bond risk premia for several economic indicators including the civilian unemployment rate (UER). Broto and Lamas (2016) use principal component analysis (PCA) to construct liquidity indices for US fixed income demonstrating the use of PCA in this domain.

While similar in motivation to the earlier work, Christopoulos (2017) takes a different modelling approach and has a different focus on CMBS. The reduced form modelling approach in Christopoulos (2017) and Christopoulos and Jarrow (2018) simulated an economy consisting of 11 forward interest rates, 48 NCREIF property x regional property indices, and 6 property specific REIT indices under risk neutral conditions. The interest rate process used a simulation using high performance computing which consisted of forward rates following Heath, Jarrow and Morton (1992) and correlated Brownian Motion for all 65 factors. The simulation implemented loan-level credit state transitions using the Cox Process as motivated by Lando (1998). The state transitions of the loans reflected the historical state transitions of 1.92 million loan life observations over the period November 2007 to December 2014. The risk adjusted loan level cashflows were first aggregated to the trust level and then allocated to the bond classes. This resulted in the capture of risk adjusted bond level cashflows. Prepayment risk was reasonably omitted in the asset pricing model due to the prepayment restrictions for commercial real estate loans which serve as collateral for CMBS. Since the simulation of the economy was conducted under risk neutral conditions, the valuation of the bond classes were also risk neutral, generating fair value prices. Since fair value prices are independent of market prices, this allowed substantial tests of market efficiency with comparisons between fair value and market prices. It also allowed a novel assessment of the composition of CMBS risks embedded within observed commercial mortgage backed securities indexed credit default swap (CMBX) risk premia through a series of transformations.

Christopoulos (2017) articulates risk components for CMBS and CMBX transforming simulated risk-neutral pricing into observable market pricing of default risk, rate risk, liquidity availability and excess liquidity availability. Sector specific innovations embedded within market pricing as found in Jarrow and Protter (2012) motivate Christopoulos and Jarrow (2018) to construct a set of common risk factors for the CMBX sector from Christopoulos

(2017) risk partitions. These values are then indexed in Christopoulos and Jarrow (2018) giving rise to the statistical rejection of the weak form of CMBS market efficiency with findings based on assessment of extraordinary returns using the ICAPM of Merton (1990). As such, those sector indexed risk partitions are well-suited to comparison with bid-ask spread measures of liquidity as estimated in this study. However, the simulated values in both prior studies were limited to monthly frequency between 2007 and 2015. This is insufficient to the task of liquidity comparison in the microstructure context in this paper. With prohibitive costs of simulation and data we secure daily risk partition risk measures corresponding to Christopoulos (2017) and Christopoulos and Jarrow (2018) by estimating from the earlier primitives as described below.

3 Daily Model

This section introduces and validates the Daily Model of risk partition estimation. The Daily Model is based on findings of the reduced form technique in the prior literature fused with daily market information. We then validate the Daily Model with data generated from four bid-ask microstructure models we implement.

3.1 Indices of CMBX

Christopoulos and Jarrow (2018) create indices of CMBX market spreads and the CMBX risk partitions introduced in Christopoulos (2017) for tractability. These indices of CMBX serve as sector-wide benchmarks for the CMBX sector. In Christopoulos and Jarrow (2018) each of the six investment grade credit tranches $k \in \{\text{AAA, AJ/AS, AA,A,BBB,BBB-}\}$there were seven different issued CMBX series, $l \in= 1, \ldots, 7$ (ie. CMBX Series 1, CMBX Series 2, ...CMBX Series 7). The prior work computed six average tranched CMBX market spreads (one for each of the six credit ratings) aggregated across seven CMBX Series, $\mathbb{S}_k(t)$,as

$$\mathbb{S}_k(t) = \frac{1}{7} \sum_{l=1}^{7} S_{kl}(t). \tag{6}$$

plus a sector level indexed CMBX market spread benchmark across all k credit ratings, computed as

$$\mathbb{S}(t) = \sum_{k=1}^{6} w_k \mathbb{S}_k(t) \tag{7}$$

where w_k is the weight of the k^{th} credit rating tranche determined by its subordination level. The weights based on subordination levels across all indices are ω_{AAA} =0.8222, $\omega_{AJ/AS}$ =0.0514, ω_{AA} =0.0514, ω_{A} =0.0257, ω_{BBB} =0.0339 and ω_{BBB-} =0.0154.

3.1.1 On-the-run time CMBX spreads

In this paper we take a different approach with the construction of a set of 'on-the-run' CMBX time series. In fixed income parlance, the most recent new issue security is considered the 'on-the-run' security and is also referred to as the 'current-coupon' as is typical in fixed-income sectors. The terminology borrows from the US Treasury vocabulary as discussed in Fleming (2001) and the residential mortgage market as discussed in Bhattacharjee and Hayre (2006). The on-the-run security is typically the most liquid of all outstanding similar issues within a sector, and coupons on securities for individual bonds (Treasuries) or tranches (within securitized capital structures) will be closest to the benchmark par/new-issue pricing for the sector. Additionally, loans underlying CMBS (which, in turn, collateralize CMBX) have interim principal payments (and sometimes losses of principal) prior to maturity as discussed in Christopoulos (2017). This is due to principal amortization and manifestations of default in the form of loss and as well as default driven prepayments. They also may exhibit restricted prepayments of principal subject to prepayment lockout and prepayment penalties. It is well-known that default and prepayment of mortgage collateral exhibit statistically positive relationships with loan age early on in the life of the loan with loans exhibiting less frequency of default or prepayment the closer they are to the origination date. As such, on-the-run issues are less likely to exhibit idiosyncratic changes in promised payment profiles the more recent the issue is to its origination date.[7]

The selection date for the on-the-run risk premium, S, with series classification, l, for credit rating class (aka 'tranche'), $k \in$ [AAA, AJ/AS, AA, A, BBB, BBB-], for all daily observation dates, t, is defined as follows:

[7]See for example Hayre, et al (1995, 2000), Christopoulos and Jarrow (2018), and Bond Market Association (2017) among others.

$$S_{lkt} = \begin{cases} l = \text{Series } 4, & t \in [11/1/2007,\ 5/22/2008] \\ l = \text{Series } 5, & t \in [5/23/2008,\ 1/24/2013] \\ l = \text{Series } 6, & t \in [1/25/2013,\ 1/26/2014] \\ l = \text{Series } 7, & t \in [1/27/2014,\ 1/25/2015] \\ l = \text{Series } 8, & t \in [1/26/2015,\ 1/24/2016] \\ l = \text{Series } 9, & t \in [1/25/2016,\ 1/24/2017] \\ l = \text{Series } 10, & t \in [1/25/2017,\ 1/29/2018] \\ l = \text{Series } 11, & t \in [1/30/2018,\ 1/28/2019] \\ l = \text{Series } 12, & t \in [1/29/2019,\ 1/31/2020] \end{cases} \tag{8}$$

For on-the-run schedule dates, the differences in observations across tranches are due to normal amortization, maturity and default driven prepayments and losses from underlying loan collateral underlying the bonds, as well as different timing of series new issuance. For example, the AAA tranches for CMBX Series 1 paid off in late 2015, while CMBX Series 12 was not issued until 1/29/2019.

Unlike Christopoulos and Jarrow (2018) we do not have the underlying loan and bond cashflow data we precludes us from simulating directly. We only have CMBX credit spreads, their changes and dates of observation. As such, resulting effective bid-ask spreads based on the microstructure models may introduce some noise and be misspecified relative to the underlying fundamental health of the collateral and market sentiment pricing. During the Great Financial Crisis, for example, over the period 5/23/2008-2/7/2013, CMBX Series 5 served as the on-the-run issue for 5 consecutive years. This is an unavoidable artefact of the market itself (and corresponding data) as *no* new CMBX Series were issued over this period.

As such, our choice to construct 'on-the-run' time series of Spreads seems appropriate. We use Eq. (8) to create the time series of CMBX market spreads and select $S_{kl}(t)$ values noted in Eq. (6) from the previous work.

3.2 On-the-run risk partitions

Additionally, each of the credit rated tranches also has a spread risk partition, S^j_{kl}, associated with each of the four CMBS risk factors $j \in \{\text{default, rates, reglq, xslq}\}$. The four indexed risk partitions indices, $\mathbb{S}^j_k(t)$,across all CMBX series, are given by

$$\mathbb{S}^j_k(t) = \frac{1}{7} \sum_{l=1}^{7} S^j_{kl}(t). \tag{9}$$

Table 4: Summary of Daily Model PCA training data

type_ticker	mean	median	min	max	variance	stdev	count
VIX	22.1251	19.3000	10.7300	69.9600	115.4094	10.7429	92
IN_DRE	15.5298	14.3950	5.6200	32.6700	25.1089	5.0109	92
IN_FR	14.4922	12.7050	2.0900	40.5800	73.8958	8.5963	92
IN_PLD	34.9227	35.5300	10.8800	64.8000	117.6452	10.8464	92
IN_SELF	3.7005	3.7800	2.3800	4.4500	0.1826	0.4273	92
LO_HST	15.6474	16.2642	3.5732	23.7400	20.1085	4.4843	92
LO_MAR	38.7738	35.6409	12.8553	82.8000	285.1512	16.8864	92
LO_WYND	18.8808	15.4538	1.5214	41.0384	125.1303	11.1862	92
LO_MGM	19.9279	13.7750	2.6200	91.7100	297.1436	17.2379	92
MF_AVB	115.1334	122.1300	43.7600	173.2800	1013.2613	31.8318	92
MF_ELS	15.9899	15.8763	6.8750	27.3750	23.7395	4.8723	92
MF_EQR	50.1745	53.9450	17.9100	77.7000	210.1242	14.4957	92
MF_UDR	22.8744	24.2100	7.9700	32.9900	32.5518	5.7054	92
OF_BXP	94.9709	101.1400	34.0400	142.2200	577.5131	24.0315	92
OF_CLI	27.6958	28.0650	14.6800	40.6800	40.0223	6.3263	92
OF_HIW	33.5559	33.2700	17.8200	46.3100	34.6371	5.8853	92
OF_SLG	76.3286	80.6000	11.1900	127.8500	809.1747	28.4460	92
OF_VNO	61.5884	61.5832	22.9945	91.3581	186.2377	13.6469	92
OT_BKD	22.6844	22.4500	3.0100	38.7200	73.2452	8.5583	92
OT_NNN	27.3768	26.4400	10.5300	42.0900	49.3787	7.0270	92
OT_PSB	62.3994	58.9550	32.3200	86.5300	175.0194	13.2295	92
OT_WPC	43.4868	38.5000	18.3400	71.2900	272.3761	16.5038	92
RT_KIM	20.9846	19.4900	7.4900	42.0900	59.1813	7.6929	92
RT_REG	47.7711	46.8450	24.3900	71.5700	124.9783	11.1794	92
RT_SPG	114.7762	108.8476	27.2503	196.5700	1909.6103	43.6991	92
RT_TCO	57.3334	58.6900	15.6000	86.9500	349.8780	18.7050	92
TSY_3MO	0.3285	0.0700	0.0050	3.8400	0.5240	0.7239	92
TSY_5YR	1.7756	1.6805	0.6140	4.0350	0.6180	0.7862	92
TSY_10YR	2.8003	2.7190	1.5040	4.3890	0.5643	0.7512	92
TSY_30YR	3.7164	3.6970	2.5560	4.7630	0.4430	0.6656	92

This table summarizes the 92 monthly observations of training data used in the Daily Model for the principal component analysis. The VIX is the CBOE volatility index. This is followed by the 25 REITs with the ticker a composite of the 6 property types industrial (IN), hotel/lodging (LO), multifamily (MF), mixed use/other (OT), office (OF) and retail (RT). Following the REITs are US Treasury yields with ticker representing the 4 maturities of 3 month, 5 year, 10 year and 30 year.

This gives us 24 indexed risk partitions, one for each of the $k \in= 1, \ldots, 6$ credit rating classes for each of $j \in= 1, \ldots, 4$ risk factors.

To create the composite CMBX sector level risk factor indices, we take the weighted across all credit ratings

$$\mathbb{S}^j(t) = \sum_{k=1}^{6} w_k \mathbb{S}_k^j(t) \tag{10}$$

where w_k is the weight of the k^{th} credit rating tranche determined by its subordination level. As with CMBX market spreads, we also create 'on-the-run' time series risk partitions which use Eq. (8) to select the time series of CMBX risk partition values, $S_{kl}^j(t)$ in Eq. (6).

3.3 Training Data

Table 4 provides a summary of the 92 training observations of the variables over the period 11/2007 thru 6/2015 that make up the economy in the Daily Model. They are used for the Daily Model estimation.

3.4 Reduction of dimension

From the training data in Table 4 we estimate the risk partitions daily with the Daily Model, over the sample period of 11/2007-4/2019. The Daily Model uses a standard linear regression on the logs of the risk partitions reported in the monthly training set of Christopoulos and Jarrow (2018) against a digest of market data (25 REITs, 4 US Treasuries, and the VIX volatility index). We simulate the daily evolution of risk components given the training set of 92 monthly observations. Throughout the estimation of the Daily Model and (later the Intraday Model), for arithmetic reasons, we use the logarithm of these values as our starting point. The 92 training observations are the indices of on-the-run CMBX spreads and their corresponding on-the-run indices of CMBX risk partitions as previously described. Although the number of observations for the training set (92) is limited, these variables have a high degree of correlation among them. It is thus possible to create a lower-dimensional set of factors which contain enough information to explain most of the variance in the original set of 30. To remove the cointegration we perform a principal components analysis (PCA), retaining enough factors n to preserve 96% of the total variance at the observed dates (in this case, 5).

For PCA in general form, let $x_q(t)$ be the value of the q-th explanatory variable at time t. For PCA loadings $p_{iq},\ i \in [1, 5], q \in [1, 30]$. Assuming, we have all 30 factors, then the elements of the i x q matrix of principal components of the observed explanatory variables $x_q(t)$ for all observed t, there exists a set of factors $f_i(t)$ such that

$$x_q(t) = \sum_{i=1}^{30} p_{iq} f_i(t) \tag{11}$$

with each explanatory variable a linear combination of the factors f_i at all times t. But we want to reduce dimension because we have only 92 observations. And so when we reduce the number of components below 30 observable variables, to 5 variables, we determine the factors with matrix multiplication as

$$f_i(t) = \sum_{q=1}^{30} p_{iq} x_q(t),\ i \in \{1...5\} \tag{12}$$

The factors $f_i(t)$ are uncorrelated[8], such that the partial sum is an unbiased estimator of $x_q(t)$,

$$x_q(t) = \sum_{i=1}^{5} p_{iq} f_i(t) + \varepsilon_n(t), \quad \mathbb{E}[\varepsilon_n] = 0 \tag{13}$$

where the error term $\mathbb{E}[\varepsilon_n] = \sum_{i=6}^{30} p_{iq} f_i(t)$, which is the minimal possible error that can be introduced in a 1:1 transformation with the technique. For highly correlated series of variables $x_q(t)$, the proportion of their internal variance explained by even small partial sums can be large.

From the covariance matrix of the explanatory variables we calculate the $(30x30)$ matrix of eigenvectors and the $(30x1)$ vector of eigenvalues. In Table 5, we report the eigenvalues all 30 principal components (Dim.1 through Dim.30). Since none of the eigenvalues are less than zero, the variance covariance matrix is said to be positive semi-definite. As the first five eigenvalues have cumulative variance of 96.11%, we are comfortable with restricting our model to the first five principal components.

Table 5: Eigenvalues

Principal Components (1:30)	eigenvalue	variance.percent	cumulative.variance.percent
Dim.1	18.1849	60.61638349	60.61638
Dim.2	7.0359	23.45291276	84.0693
Dim.3	1.7472	5.82397743	89.89327
Dim.4	1.2534	4.177832953	94.07111
Dim.5	0.6139	2.046302198	96.11741
Dim.6	0.3176	1.058805619	97.17621
Dim.7	0.2173	0.724332503	97.90055
Dim.8	0.1257	0.419073426	98.31962
Dim.9	0.1012	0.337476267	98.6571
Dim.10	0.0853	0.284225276	98.94132
Dim.11	0.0647	0.215505035	99.15683
Dim.12	0.0444	0.148149508	99.30498
Dim.13	0.0387	0.128858723	99.43384
Dim.14	0.0341	0.113664094	99.5475
Dim.15	0.0223	0.0744866	99.62199
Dim.16	0.0174	0.058036274	99.68002
Dim.17	0.0154	0.051404095	99.73143
Dim.18	0.0132	0.043836793	99.77526
Dim.19	0.0122	0.040651317	99.81591
Dim.20	0.0115	0.038209358	99.85412
Dim.21	0.0105	0.034944086	99.88907
Dim.22	0.0080	0.026785645	99.91585
Dim.23	0.0061	0.020269155	99.93612
Dim.24	0.0044	0.014599285	99.95072
Dim.25	0.0043	0.014407187	99.96513
Dim.26	0.0035	0.011698808	99.97683
Dim.27	0.0028	0.009360687	99.98619
Dim.28	0.0024	0.007873984	99.99406
Dim.29	0.0012	0.003918992	99.99798
Dim.30	0.0006	0.002018451	100

We thus transform our 30 variables into a *digest* of just 5 variables with $x_q(t)$ the value

[8] See the proof of this in Appendix A.4.

of the q-th economic variable at time t. The PCA loadings (aka 'rotations') are summarized in Table 6.

Table 6: Principal component loadings

Type	PropType	Ticker	Factor Name	PC1	PC2	PC3	PC4	PC5
VIX	NA	VIX	VIX	0.1748	0.0107	-0.3148	-0.2332	-0.4369
REIT	Industrial	DRE	IN_DRE	-0.1406	-0.2886	-0.097	-0.0937	-0.0245
REIT	Industrial	FR	IN_FR	-0.1334	-0.2833	-0.1556	-0.1863	0.1269
REIT	Industrial	PLD	IN_PLD	-0.1622	-0.2605	0.0067	-0.1089	0.1225
REIT	Industrial	SELF	IN_SELF	-0.1163	-0.0726	0.5916	-0.1331	0.1507
REIT	Hotel	HST	LO_HST	-0.2184	-0.0537	0.0704	0.1801	-0.0047
REIT	Hotel	MAR	LO_MAR	-0.2091	0.0519	-0.1853	0.2272	-0.1395
REIT	Hotel	WYND	LO_WYND	-0.2161	0.1097	-0.122	0.107	0.1364
REIT	Hotel	MGM	LO_MGM	-0.0578	-0.3335	-0.1355	-0.1244	0.1721
REIT	Multifamily	AVB	MF_AVB	-0.2207	0.0755	0.1151	-0.0826	-0.1916
REIT	Multifamily	ELS	MF_ELS	-0.2157	0.1168	-0.0943	0.0968	-0.0964
REIT	Multifamily	EQR	MF_EQR	-0.2135	0.1001	0.1302	-0.0469	-0.3028
REIT	Multifamily	UDR	MF_UDR	-0.2201	-0.0111	0.0938	-0.0807	-0.3541
REIT	Office	BXP	OF_BXP	-0.2296	-0.0055	0.0615	-0.0646	-0.1693
REIT	Office	CLI	OF_CLI	0.0629	-0.2369	0.514	-0.0331	-0.1139
REIT	Office	HIW	OF_HIW	-0.2195	-0.0137	0.0278	0.2056	-0.089
REIT	Office	SLG	OF_SLG	-0.2249	-0.0927	-0.0024	0.0316	-0.068
REIT	Office	VNO	OF_VNO	-0.2131	-0.0941	0.0421	0.1063	-0.3489
REIT	Mixed/Other	BKD	OT_BKD	-0.2124	-0.0833	-0.0401	0.2277	0.1163
REIT	Mixed/Other	NNN	OT_NNN	-0.2213	0.0892	-0.0338	0.0709	0.1337
REIT	Mixed/Other	PSB	OT_PSB	-0.2124	0.0861	-0.0613	0.1464	0.2303
REIT	Mixed/Other	WPC	OT_WPC	-0.2079	0.1098	-0.1229	0.0496	0.3096
REIT	Retail	KIM	RT_KIM	-0.116	-0.3012	-0.0986	-0.2131	-0.0795
REIT	Retail	REG	RT_REG	-0.1783	-0.2091	-0.1494	-0.1061	-0.0406
REIT	Retail	SPG	RT_SPG	-0.2247	0.0994	-0.0166	-0.0345	0.0524
REIT	Retail	TCO	RT_TCO	-0.2183	0.0611	0.1079	-0.1874	0.1506
Treasury	NA	3MO	TSY_3MO	0.0137	-0.347	-0.0736	-0.2146	0.1305
Treasury	NA	5YR	TSY_5YR	0.0803	-0.311	-0.1406	0.3288	-0.1094
Treasury	NA	10YR	TSY_10YR	0.1142	-0.2744	-0.0077	0.4121	-0.0729
Treasury	NA	30YR	TSY_30YR	0.1228	-0.2373	0.1884	0.4231	0.0084

This table summarizes the principal component loadings (rotations). The first column indicates the type of the variable (REIT, VIX, or Treasury). The second column indicates the property type which is indicated for REITs and not applicable (NA) for the other variables. The third column provides the ticker symbol for the REITs and indication of VIX or the maturity for the Treasuries. The fourth column is the internal data name of the variable. The fifth through ninth columns (PCA1, ... PCA5) indicates the principal component loadings determined from the eigenvalues and eigenvectors.

3.5 Initial conditions

This five-dimensional digest in Table 6 is used to construct our factor volatility explanatory variables (described below) for the risk component estimates. The factor volatilities f_{it} are a function of the PCA loadings in Table 6 and the explanatory variables observed at time t. Let R^Tbe the $5x30$ PCA loadings matrix, and E^T the $30x92$ matrix of the 30 variables over 92 observation dates, whose elements are $x_q(t)$. Then the factor matrix, F, calculated using matrix multiplication as the product of R^T and E^T,

$$\mathrm{F} = \mathrm{R}^T\mathrm{E}^T = \begin{bmatrix} f_{1,1} & \cdots & f_{t,5} \\ \vdots & \ddots & \vdots \\ f_{92,1} & \cdots & f_{92,5} \end{bmatrix} \tag{14}$$

Table 7: Summary of simulated risk partitions as dependent variables for OLS

type_ticker	mean	median	min	max	variance	stdev	count
market spread AAA	118.1295	99.9420	25.0000	550.7100	8084.8459	89.9158	92
market spread AJ	473.0770	390.9575	55.1621	1703.7300	81736.2628	285.8955	92
market spread AA	981.0831	1000.9948	76.1364	2525.8600	261721.8298	511.5876	92
market spread A	1555.3875	1609.9502	89.0328	3667.6336	604157.4778	777.2757	92
market spread BBB	3053.6276	2703.6012	162.1271	10491.7654	3845199.4134	1960.9180	92
market spread BBBm	4252.5810	3952.0150	207.4319	14720.7284	8199398.1483	2863.4591	92
def_AAA	8.0555	1.6607	0.0277	195.8856	790.1596	28.1098	92
rates_AAA	52.8702	37.4083	1.6727	263.6903	2712.6277	52.0829	92
reglq_AAA	57.2037	44.8207	6.7286	262.8916	2294.4850	47.9008	92
xslq_AAA	134.1662	134.5824	0.0000	294.3972	8930.3539	94.5005	92
def_AJ	56.0297	3.3434	0.0488	1146.3424	42838.9678	206.9758	92
rates_AJ	211.9356	164.1067	0.5000	1251.9229	37946.1184	194.7976	92
reglq_AJ	200.6518	221.3317	0.0000	432.8215	10881.9687	104.3167	92
xslq_AJ	0.5256	0.0000	0.0000	22.5208	9.9243	3.1503	92
def_AA	284.7420	159.6041	30.4583	1735.0750	107892.5071	328.4699	92
rates_AA	561.9648	617.7424	0.0000	1506.2897	111677.9404	334.1825	92
reglq_AA	134.3763	123.8860	0.0000	385.7624	13602.5250	116.6299	92
xslq_AA	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	92
def_A	577.6416	535.7316	3.2881	1959.6544	170249.2845	412.6128	92
rates_A	721.9083	708.1810	0.0000	1788.8356	293804.1400	542.0370	92
reglq_A	255.8375	0.0000	0.0000	2564.8621	210292.7394	458.5769	92
xslq_A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	92
def_BBB	965.0640	740.0350	7.5201	2687.1501	710113.0550	842.6821	92
rates_BBB	1180.3222	920.4655	0.0000	5171.6783	1298837.7798	1139.6656	92
reglq_BBB	908.2413	0.0000	0.0000	5811.4664	1878471.1005	1370.5733	92
xslq_BBB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	92
def_BBBm	2583.6984	2682.1252	207.4319	4683.3151	1266031.2743	1125.1806	92
rates_BBBm	527.5114	0.0000	0.0000	6610.2034	1342570.6397	1158.6935	92
reglq_BBBm	1141.3713	0.0000	0.0000	7660.1845	4407188.9990	2099.3306	92
xslq_BBBm	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	92

This table summarizes the 92 monthly observations of the markets spreads (mktsprd) and the simulated risk partitions indexed in Christopoulos and Jarrow (2018) in spread (bps) form. As noted in Eq. (15), the market spreads in this table serve as independent variables while the simulated risk partitions serves as dependent variables. The ticker is a composite of the 4 types of risk partitions default (def), interest rates (rates), liquidity (reglq) and excess liquidity (xslq) combined with the credit rating class names of AAA, AJ, AA, A, BBB and BBB- (BBBm).

yields a $92x5$ matrix, the elements of which are the factors, f_{ti} that we use to capture the volatility in our model. We switch the notation $f_{ti} \equiv f_{it}$for the remaining calculations. The factor volatility we determine from the PCA is $v_i(t) \equiv [f_i(t) - f_i(t-1)]^2$. For each month, t, to estimate the risk partition we begin with an opening value based on the previous two one-month volatilities of the variables $v_i(t)$ and $v_i(t-1)$ and the CMBX indexed market spread $\mathbb{S}_k(t)$ for each credit rating class, as these are the values available to us for training purposes. Table 7 provides a summary of the market spreads used as independent variables and the risk partitions used as dependent variables for the OLS. The risk partitions are the proportional results from the 92 monthly simulations and indexing of Christopoulos and Jarrow (2018) over the period 11/2007 thru 6/2015. The initial risk component $y_{jk}(t_0)$, $j \in \{\text{def}, \text{rate}, ...\}, k \in \{\text{AAA}, \text{AJ/AS}, ...\}$, is given by

$$y_{jk}(t) = \alpha_{jk} + \sum_{i=1}^{5} \beta_{ijk} v_i(t) + \gamma_{ijk} v_i(t-1) + \delta_{ijk} \left[v_i(t) - v_i(t-1)\right]^2 + \psi_k \mathbb{S}_k(t) + \varepsilon_{jk}(t) \quad (15)$$

Table 8: OLS of default risk partitions with factor volatilities

	defAAA	defAJ	defAA	defA	defBBB	defBBBm	defALL
alpha	-10.7767*	-61.8278.	22.6571	590.1731***	993.1729***	121.6843	50.5395***
	(4.1368)	(35.8692)	(74.1767)	(108.5017)	(102.68)	(115.3387)	(12.9274)
vPC1	6.5726**	43.0815**	26.1433	2.574	40.8001	86.041*	10.6503*
	(2.2568)	(14.8203)	(25.8927)	(36.5177)	(30.0269)	(34.8689)	(4.4446)
vPC2	-10.9708.	-45.7803	-26.6861	-26.8434	-209.1309*	-210.8679*	-20.9231.
	(6.3299)	(41.1619)	(72.0341)	(103.1963)	(86.2198)	(99.733)	(12.3307)
vPC3	12.4146	145.5318	540.8874***	901.7894***	682.7787***	539.3425**	111.771***
	(13.7431)	(90.7345)	(146.2545)	(196.6739)	(161.0751)	(186.1063)	(24.0677)
vPC4	-12.5255	-140.2799*	-116.8911	37.1549	140.519	-140.3293	-20.232
	(10.4518)	(68.8229)	(118.749)	(166.813)	(137.3145)	(159.1696)	(20.0894)
vPC5	-9.6894	21.136	201.2984	-91.5091	-327.7452	133.0946	-14.9287
	(19.4001)	(127.7155)	(221.7927)	(311.7747)	(259.3555)	(300.3334)	(37.4276)
vm1PC1	5.7167*	25.5661	6.1495	3.3371	27.5806	61.3883	7.0571
	(2.4355)	(16.0857)	(28.2306)	(39.6899)	(32.4783)	(37.6898)	(4.8548)
vm1PC2	-1.9859	11.382	66.1957	15.3863	-129.5663.	-94.7728	0.3806
	(5.7632)	(37.0799)	(65.329)	(93.4886)	(77.6678)	(90.0697)	(11.1128)
vm1PC3	-28.6259**	-59.0789	271.0683*	504.6081***	331.9641**	65.3341	22.4011
	(10.0256)	(63.9269)	(103.9853)	(145.0704)	(119.445)	(139.1512)	(17.4612)
vm1PC4	-13.4965	-80.9366	-77.608	88.963	104.68	-96.5376	-16.9641
	(10.9315)	(71.8723)	(124.0292)	(174.2153)	(143.3167)	(166.195)	(20.9625)
vm1PC5	5.2977	68.0332	307.9458	77.2959	-316.3798	142.7753	11.1588
	(22.9842)	(150.0821)	(258.6374)	(363.3975)	(300.6296)	(348.8222)	(43.7195)
vvm1PC1	0.1493	0.9124	1.6063	3.2727.	3.2997*	2.1411	0.5178*
	(0.1067)	(0.7051)	(1.2157)	(1.702)	(1.4014)	(1.6201)	(0.2048)
vvm1PC2	0.6574	-3.3348	-10.3103	-13.4763	0.9685	4.0089	-0.8307
	(1.0164)	(6.5063)	(11.1916)	(15.8043)	(13.0475)	(15.1449)	(1.8922)
vvm1PC3	-1.2238	-16.1888	-65.566**	-115.4626***	-86.9145***	-69.1465**	-13.729***
	(1.8167)	(12.0746)	(19.4756)	(26.1569)	(21.416)	(24.7358)	(3.1985)
vvm1PC4	4.6921	81.5522*	76.0035	20.7611	57.2081	174.8484*	17.5748.
	(4.9887)	(32.829)	(56.7403)	(79.8132)	(65.7941)	(76.3209)	(9.6051)
vvm1PC5	-27.798	-179.6284	-526.2122*	-607.3429.	-58.3076	-413.3655	-88.5667*
	(22.5853)	(144.9092)	(245.7735)	(344.6383)	(285.52)	(331.9571)	(41.4507)
mktsprd_k	0.1318***	0.1184*	0.051	-0.0474	0.1798***	0.3498***	0.1496***
	(0.0258)	(0.0501)	(0.0464)	(0.0382)	(0.0215)	(0.0196)	(0.0298)
F-test	13.46	7.108	5.59	3.91	13.76	36.07	12.94
	0	0	0	0	0	0	0
Adj. Rsq	0.6914	0.5234	0.4521	0.3435	0.6964	0.8631	0.6822

Table 9: OLS of rates risk partitions with factor volatilities

	ratesAAA	ratesAJ	ratesAA	ratesA	ratesBBB	ratesBBBm	ratesALL
alpha	0.7599	14.7093	-56.8643	-610.7355***	-1370***	-727.7179***	-37.0003***
	(2.3686)	(27.5605)	(75.5302)	(117.5056)	(151.7)	(154.6173)	(7.4369)
vPC1	1.2789	-22.3523.	3.1219	12.529	-8.244	-24.0701	-4.5644.
	(1.2922)	(11.3873)	(26.3652)	(39.5481)	(44.38)	(46.7435)	(2.5569)
vPC2	-11.0221**	2.2831	-65.0849	-19.3482	151.7	103.2575	1.9126
	(3.6243)	(31.6272)	(73.3485)	(111.76)	(127.4)	(133.6971)	(7.0936)
vPC3	3.5193	-32.378	-504.5734**	-852.9238***	-1019***	-985.9521***	-50.3539***
	(7.8689)	(69.7169)	(148.9232)	(212.9947)	(238)	(249.4849)	(13.8457)
vPC4	12.9372*	98.5707.	94.672	8.6312	-109.9	34.8594	15.5986
	(5.9844)	(52.8809)	(120.9158)	(180.6559)	(202.9)	(213.3749)	(11.557)
vPC5	-8.1576	36.4812	-92.4278	216.7334	650.	284.9899	-2.5607
	(11.1079)	(98.1316)	(225.8397)	(337.6472)	(383.3)	(402.612)	(21.5314)
vm1PC1	1.0172	-8.2242	16.5593	13.804	-1.232	-26.5671	-3.0305
	(1.3945)	(12.3597)	(28.7457)	(42.9835)	(48)	(50.525)	(2.7929)
vm1PC2	-6.8769*	-36.4399	-109.1081	-57.646	73.94	50.1415	-3.653
	(3.2998)	(28.4907)	(66.521)	(101.2467)	(114.8)	(120.743)	(6.393)
vm1PC3	-0.9608	58.8951	-296.5544**	-509.8478**	-583.4**	-407.5405*	-20.991*
	(5.7403)	(49.119)	(105.8827)	(157.109)	(176.5)	(186.5392)	(10.0451)
vm1PC4	4.9163	78.5986	73.4133	-4.9955	-38.09	20.884	6.1253
	(6.259)	(55.2239)	(126.2923)	(188.6725)	(211.8)	(222.7928)	(12.0593)
vm1PC5	-14.7768	7.9833	-197.0646	39.5006	540	113.6629	-15.0279
	(13.16)	(115.3173)	(263.3566)	(393.5539)	(444.3)	(467.6137)	(25.151)
vvm1PC1	0.0954	-0.3552	-1.4311	-3.043	-4.396*	-3.5722	-0.1345
	(0.0611)	(0.5418)	(1.2379)	(1.8433)	(2.071)	(2.1719)	(0.1178)
vvm1PC2	0.5601	3.7786	14.6394	15.4617	5.816	3.5045	1.1716
	(0.5819)	(4.9992)	(11.3958)	(17.1159)	(19.28)	(20.3025)	(1.0885)
vvm1PC3	-0.1488	2.6962	63.071**	109.5375***	132***	131.8938***	6.3621***
	(1.0402)	(9.2776)	(19.8309)	(28.3275)	(31.65)	(33.1596)	(1.84)
vvm1PC4	-5.9098*	-54.4616*	-50.1199	-33.0714	-78.64	-116.4152	-14.7355**
	(2.8564)	(25.2245)	(57.7756)	(86.4365)	(97.24)	(102.312)	(5.5256)
vvm1PC5	10.0181	95.6128	498.4666.	518.6177	131.5	561.9252	40.7162.
	(12.9316)	(111.3426)	(250.2581)	(373.238)	(422)	(445.0051)	(23.8458)
mktsprd_k	0.3738***	0.4311***	0.6047***	0.7357***	0.6704***	0.5004***	0.5991***
	(0.0148)	(0.0385)	(0.0473)	(0.0413)	(0.0318)	(0.0262)	(0.0172)
F-test	116.9	15.29	15.38	28.31	38.92	33.63	103
	0	0	0	0	0	0	0
Adj. Rsq	0.9542	0.7197	0.7211	0.8308	0.8721	0.8544	0.9483

Table 10: OLS of liquidity risk partitions with factor volatilities

	reqlqAAA	reqlqAJ	reqlqAA	reqlqA	reqlqBBB	reqlqBBBm	reqlqALL
alpha	18.4029***	118.9706***	1.7014	-313.2145***	-612.6711***	-688.2123***	-13.5367.
	(4.2406)	(19.9913)	(44.0622)	(88.309)	(111.1486)	(80.0271)	(7.6505)
vPC1	-4.8539*	-3.8118	3.0006	20.6221	7.3018	-25.764	-6.0858*
	(2.3134)	(8.2599)	(15.3807)	(29.7215)	(32.5034)	(24.1936)	(2.6304)
vPC2	13.5444*	-8.5059	-16.3051	-37.9445	49.0844	131.2454.	19.0093*
	(6.4887)	(22.9411)	(42.7894)	(83.9909)	(93.3309)	(69.1991)	(7.2974)
vPC3	-12.5931	-145.3003**	-385.2747***	-603.8999***	-510.9596**	-411.5784**	-61.4267***
	(14.0879)	(50.5699)	(86.8776)	(160.0719)	(174.36)	(129.1287)	(14.2434)
vPC4	6.6246	62.5189	35.5547	-52.19	-91.4001	29.2656	4.6369
	(10.714)	(38.3577)	(70.5389)	(135.7683)	(148.6397)	(110.4389)	(11.889)
vPC5	6.8217	-93.1821	-64.7353	110.5509	407.0535	489.4531*	17.4923
	(19.8867)	(71.1808)	(131.7486)	(253.7519)	(280.7463)	(208.3845)	(22.1499)
vm1PC1	-2.9833	1.9172	7.891	12.1455	11.1894	-2.9167	-4.0258
	(2.4966)	(8.9652)	(16.7695)	(32.3034)	(35.157)	(26.1508)	(2.8731)
vm1PC2	0.8198	-34.084	-53.6298	-32.8496	23.6343	51.3935	3.2702
	(5.9078)	(20.666)	(38.8065)	(76.0899)	(84.0736)	(62.4943)	(6.5766)
vm1PC3	27.6438**	-43.2358	-242.5588***	-367.3343**	-300.0991*	-198.4602*	-1.4145
	(10.2771)	(35.629)	(61.769)	(118.0721)	(129.2964)	(96.5492)	(10.3337)
vm1PC4	16.0564	38.2344	32.2554	-65.7412	-55.6258	60.6023	10.8392
	(11.2057)	(40.0572)	(73.6754)	(141.793)	(155.137)	(115.3134)	(12.4057)
vm1PC5	-1.2835	-109.2202	-121.3679	-9.8251	353.6387	509.7061*	3.8662
	(23.5607)	(83.6466)	(153.6349)	(295.7674)	(325.4245)	(242.0282)	(25.8734)
vvm1PC1	-0.2354*	-0.379	-1.0069	-2.0164	-2.2893	-2.2005.	-0.3834**
	(0.1094)	(0.393)	(0.7221)	(1.3853)	(1.517)	(1.1241)	(0.1212)
vvm1PC2	-0.7587	1.7791	7.6116	10.2662	4.0287	0.0264	-0.3405
	(1.0419)	(3.6262)	(6.648)	(12.8631)	(14.1237)	(10.5082)	(1.1198)
vvm1PC3	1.0583	17.6926*	48.6955***	79.6236***	67.6857**	52.3923**	7.3681***
	(1.8623)	(6.7296)	(11.5688)	(21.289)	(23.1823)	(17.1628)	(1.8929)
vvm1PC4	1.4512	-14.8557	-11.3304	14.7181	-20.0621	-128.8652*	-2.8393
	(5.1139)	(18.2969)	(33.7047)	(64.9596)	(71.2205)	(52.9548)	(5.6844)
vvm1PC5	30.8461	186.1648*	319.9671*	399.533	41.9613	-157.6773	47.8575.
	(23.1518)	(80.7635)	(145.9936)	(280.4994)	(309.0687)	(230.3264)	(24.5308)
mktsprd_k	0.1689***	0.1252***	0.2053***	0.2918***	0.221***	0.2394***	0.2513***
	(0.0265)	(0.0279)	(0.0276)	(0.0311)	(0.0233)	(0.0136)	(0.0177)
F-test	12.84	3.418	6.791	9.217	8.12	25.98	15.66
	0	0.0002	0	0	0	0	0
Adj. Rsq	0.6803	0.303	0.51	0.5963	0.5614	0.8179	0.725

Table 11: OLS of excess liquidity risk partitions with factor volatilities

	xslqAAA	xslqAJ	xslqAA	xslqA	xslqBBB	xslqBBBm	xslqALL
alpha	113.6527***	0.3828*	0.1516***	0.193***	0.2314***	0.1223.	17.3036
	(9.0347)	(0.1481)	(0.0196)	(0.0173)	(0.0178)	(0.0651)	(11.2882)
vPC1	9.3319.	-0.0308	-0.0108	-0.0074	-0.0063	-0.0306	0.4606
	(4.9288)	(0.0612)	(0.0068)	(0.0058)	(0.0052)	(0.0197)	(3.881)
vPC2	-22.0347	-0.0215	0.0403*	0.023	0.0115	0.0585	-11.4069
	(13.8245)	(0.1699)	(0.019)	(0.0165)	(0.0149)	(0.0563)	(10.7671)
vPC3	-0.4144	0.4244	0.0889*	0.0736*	0.068*	0.0254	-85.719***
	(30.0148)	(0.3746)	(0.0386)	(0.0314)	(0.0279)	(0.1051)	(21.0158)
vPC4	-7.3099	-0.2611	-0.0178	-0.0141	-0.0093	-0.0566	-12.0873
	(22.8266)	(0.2842)	(0.0313)	(0.0267)	(0.0238)	(0.0899)	(17.542)
vPC5	-55.9739	0.0404	0.0289	0.0101	-0.0232	0.1019	-31.214
	(42.3695)	(0.5273)	(0.0585)	(0.0498)	(0.0449)	(0.1696)	(32.6817)
vm1PC1	9.2152.	-0.0497	-0.0097	-0.0056	-0.0051	0.0077	-3.0403
	(5.3191)	(0.0664)	(0.0074)	(0.0063)	(0.0056)	(0.0213)	(4.2392)
vm1PC2	-13.8536	0.0573	0.0359*	0.0204	0.0121	-0.009	-8.2156
	(12.5868)	(0.1531)	(0.0172)	(0.0149)	(0.0135)	(0.0509)	(9.7037)
vm1PC3	-6.8977	0.3309	0.0677*	0.0476*	0.0431*	-0.0252	-49.0566**
	(21.8958)	(0.2639)	(0.0274)	(0.0232)	(0.0207)	(0.0786)	(15.2471)
vm1PC4	-20.5648	-0.4027	-0.0238	-0.0197	-0.0171	0.0589	-16.1122
	(23.8742)	(0.2967)	(0.0327)	(0.0278)	(0.0248)	(0.0939)	(18.3044)
vm1PC5	-31.2948	-0.1524	0.0488	0.0375	-0.0012	0.3677.	-52.344
	(50.1971)	(0.6197)	(0.0682)	(0.0581)	(0.0521)	(0.197)	(38.1757)
vvm1PC1	0.158	0.0011	0.0001	0.0001	0.0002	0.0002	-0.0722
	(0.2331)	(0.0029)	(0.0003)	(0.0003)	(0.0002)	(0.0009)	(0.1789)
vvm1PC2	-0.6102	0.0014	-0.0033	-0.0018	-0.0009	0.001	1.3111
	(2.2197)	(0.0269)	(0.003)	(0.0025)	(0.0023)	(0.0086)	(1.6522)
vvm1PC3	-0.0805	-0.0589	-0.0123*	-0.0103*	-0.0096*	-0.0035	11.0896***
	(3.9677)	(0.0499)	(0.0051)	(0.0042)	(0.0037)	(0.014)	(2.7929)
vvm1PC4	16.1625	0.1147	-0.0036	0.0003	0.0038	-0.018	9.1813
	(10.8953)	(0.1355)	(0.015)	(0.0128)	(0.0114)	(0.0431)	(8.3871)
vvm1PC5	39.3148	0.0093	-0.1213.	-0.0907	-0.0515	-0.2458	95.7047*
	(49.326)	(0.5983)	(0.0648)	(0.0551)	(0.0495)	(0.1875)	(36.1947)
mktsprd_k	-0.2838***	-0.0002	0***	0***	0***	0	0.1644***
	(0.0564)	(0.0002)	(0)	(0)	(0)	(0)	(0.0261)
F-test	5.556	0.2605	2.654	5.419	7.988	0.4475	7.052
	0	0.9979	0.0025	0	0	0.9631	0
Adj. Rsq	0.4502	-0.1533	0.2292	0.4427	0.5568	-0.1103	0.5211

The coefficients $\{\alpha_{jk}, \beta_{ijk}, \gamma_{ijk}, \delta_{ijk}, \psi_k\}$ are determined through OLS by minimizing the sum of the squared error $\sum_t \varepsilon_{jk}(t)$ with t indexed in months with $\mathbb{E}[\varepsilon_{jk}(t)] = 0$. In total, 16 coefficients are estimated, one for each of the 15 separate volatility components, and one for the indexed market spread corresponding to the credit rating class. Tables 8, 9, 10, and 11 capture the results of the 28 OLS in Eq (15). The OLS capture 90 monthly observations over the period 12/2007 - 6/2015. The dependent variables captured default, rates, liquidity and excess liquidity risk partitions as simulated and indexed in Christopoulos (2017) and Christopoulos and Jarrow (2018), respectively. The alpha and independent variables are listed in the first column, with vPC# capturing $v_i(t)$, vm1PC# capturing $v_i(t-1)$, and vvm1PC# capturing $[v_i(t) - v_i(t-1)]^2$ volatilities for the i-th principal components, and mktsprd_k capturing the market spread for the k-th credit rating. The four tables are organized by j-th risk partition and capture $k = 6$ investment grade ratings classes and the indexed aggregation across all credit ratings classes. 26 of 28 regressions are significant as measured by the F-test. Adjusted R-squared values range from -0.11 to 0.95. Generally, the market spread is highly significant, but not always. The volatility factors also exhibit instances of significance from 0.10% to 10.00% as well as many instances of insignificance. The third principal component for all three volatilities $v_i(t)$, $v_i(t-1)$, and $[v_i(t) - v_i(t-1)]^2$ appears to be consistently more significant across all regressions compared with other principal components. Recall, our primary purpose in this portion of the paper is to synthetically increase the frequency of simulated risk partitions, without simulating, in a systematic way. We do this by capturing changes in correlated variables and relating them to risk partitions. As such, these results are acceptable to that task and are used to estimate the risk partitions daily, as discussed below.

After determining the estimates in Eq. (15) we then predict the daily spread risk decompositions using Eq. (16) combining the estimates and 2828 daily observations . We adjust the lookback of the factor volatilities with 22 trading days equal to one month from the date of the daily observations for the updated calculations. We re-express the estimation model for the daily predicted model as follows. For all trading days, u, we compute predicted values on the left hand side based on the PCA load factors and estimated coefficients on the right hand side as is given by

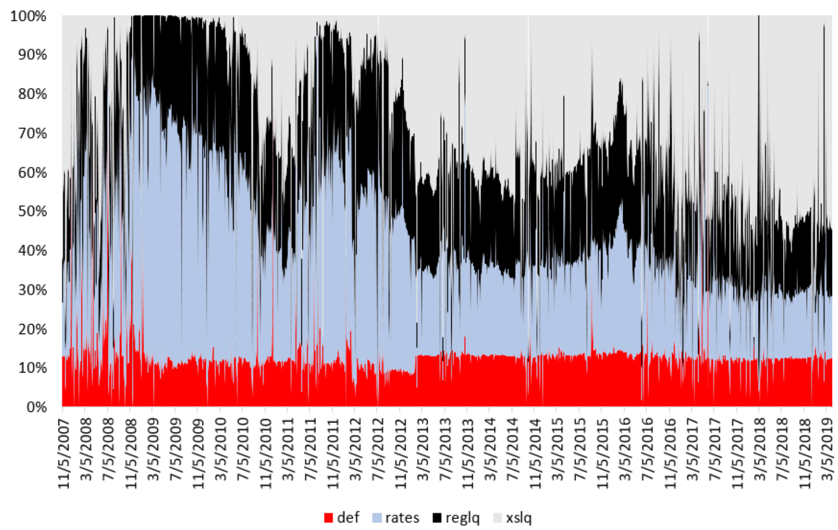
$$\hat{y}_{jk}(u) = \alpha_{jk} + \sum_{i=1}^{5} \beta_{ijk} v_i(u) + \gamma_{ijk} v_i(u-22) + \delta_{ijk} \left[v_i(u) - v_i(u-22)\right]^2 + \psi_k \mathbb{S}_k(t) \qquad (16)$$

with the final risk composition then computed as a proportion of the total for the bond:

$$\bar{y}_{jk}(u) = \frac{\hat{y}_{jk}(u)}{\sum_j \hat{y}_{jk}(u)} \qquad (17)$$

We calculate all four risk components as the proportions of Eq. (17) for the CMBX sector across all credits in Figure 1.

Figure 1: Daily Indexed CMBX Risk Partitions



This figure depicts the estimated risk partitions defined in Eq. (17) for all four risk components of default, interest rates, liquidity and excess liquidity on a daily basis in the plot on the left from 11/2007 thru 4/2019. These estimates are based on the monthly training set of 92 weighted observations of risk partitions for the CMBX sector overall across all credits as depicted in Fig. (9) in Christopoulos and Jarrow (2018). The x-axis capture the trading days over the sample period and the y-axis captures the proportions of risk.

The time series show default, interest rate, liquidity availability and excess liquidity availability indices for the CMBX as introduced and defined in Christopoulos (2017). Since this paper is focused on liquidity for CMBX and REITs, as measured by effective bid-ask spreads compared with estimated reduced form liquidity measures, we extract the values for liquidity and excess liquidity depicted in Figure 1 and use them as independent variables in the evaluation of CMBX liquidity.

3.6 Validation of the Daily Model

To validate the daily estimation approach, we use the effective bid/ask spreads determined from the microstructure models discussed below as measures of CMBX liquidity from the perspective of the market and compare to the perspective revealed by the risk partitions. In this section we restrict the analysis to AAA and BBB[9] CMBX to correspond to the AAA and BBB corporate bond spreads used in our analysis. We incorporate that data with the daily estimated reduced form liquidity estimates from the Daily Model. We use ordinary least squares (OLS), vector autoregressive (VAR), Granger causality (Granger) and impulse response function (IRF) techniques. These analyses allow us to assess the liquidity of the CMBX sector with our approach, historically and to validate the Daily Model approach. The explanatory factors driving bid-ask spreads we consider are US Treasuries, corporate bond spreads, the VIX volatility index and the exogenously determined reduced form liquidity indices for the CMBX sector.

3.6.1 Microstructure models to generate Daily Model validation data

Following, Christopoulos (2020), we implement the microstructure models applied to CMBX. In all cases, we substitute the daily mark-to-market spread, S_t, for P_t for our estimate of the bid-ask spreads of mid-market spreads as permitted by i.i.d. Thus, in all microstructure models implemented below we are computing bid/ask spreads of spreads (as discussed in Appendix A.1) in keeping with the way in which such fixed income spread products are traded.

In Model 1 (Roll), we implement Roll (1984) for end-of-day mid-market spread risk premia. We adjust the price notation P_t notation in Eq. (1) to accommodate bid-ask spreads of risk premia (bid-ask spreads of Spreads), S_t, such that

$$s_t = 2\sqrt{-\mathrm{COV}(\Delta S_t, \Delta S_{t-1})} \tag{18}$$

To address positive autocovariance in the implementation we follow Harris (1990) in Eq. (2)

[9]BBB CMBX are split rated based on selection for CMBX 1 thru 5 of BBB and for CMBX 6-12 BBB-. In this section, we simply refer to that time series at BBB for brevity.

$$s_t = \begin{cases} s_t^+ = -2\sqrt{\mathrm{COV}(\Delta S_t, \Delta S_{t-1})} & \text{for } \mathrm{COV}(\Delta S_t, \Delta S_{t-1}) > 0 \\ s_t^- = 2\sqrt{-\mathrm{COV}(\Delta S_t, \Delta S_{t-1})} & \text{for } \mathrm{COV}(\Delta S_t, \Delta S_{t-1}) \leq 0 \end{cases} \tag{19}$$

which preserves the sign of autocovariance resulting in negative bid-ask spreads.

In Model 2 (Restricted Roll) we restrict Model 1, by simply dropping (or 'zeroing' in depictions, not statistics) the observations with positive autocovariance as suggested by Hasbrouck (2009) and Foucault, Pagano and Roell (2013).

In Model 3 (Absolute Roll), the Absolute Roll Measure for risk premia (aka 'Spreads') is given by

$$s_t = 2\sqrt{\left|-\mathrm{COV}\left(\Delta S_t, \Delta S_{t-1}\right)\right|} \tag{20}$$

following the derivations of Eq. (5) and Eq. (47).

In Model 4 (Thompson and Waller), we implement the model of Thompson and Waller (1987) and restate their absolute value of 5-day moving average *price* changes to 5-day moving average of changes in mid-market credit risk premia (aka 'Spreads') defined as

$$\overline{|\Delta S_t|} = \frac{1}{5}\sum_{t=1}^{5} |\Delta S_t| \tag{21}$$

with S_t the observed mid-market credit spread at time t. The bid-side spread is given by

$$B_t = S_t + \left(\frac{\overline{|\Delta S_t|}}{2}\right) \tag{22}$$

and the ask-side spread[10] is given by

$$A_t = \max\left(0,\ S_t - (B_t - S_t)\right) \tag{23}$$

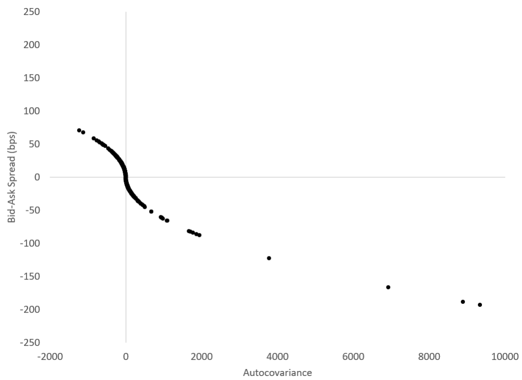
such that the bid-ask spread, s_t, of mid-market fixed income Spreads is given by

$$s_t = B_t - A_t \tag{24}$$

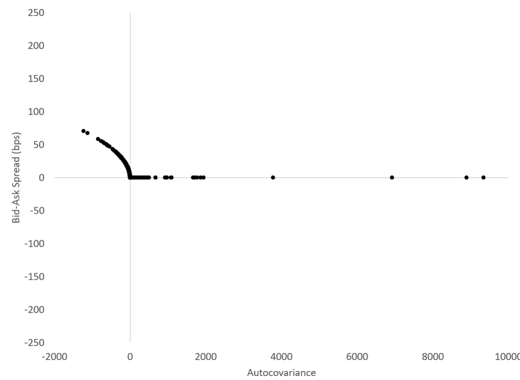
[10]The lower boundary of zero ensures offer side spreads cannot be negative. The boundary is never reached in this study.

3.6.2 Validation of Daily Model bid-ask spreads

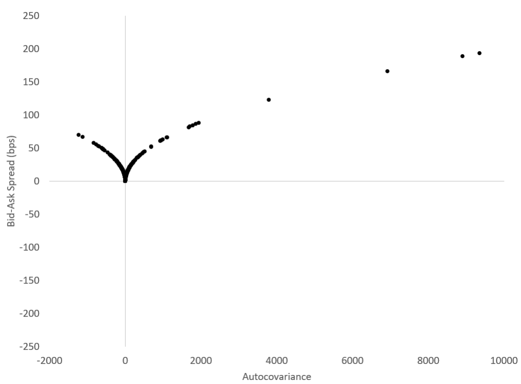
Figure 2: AAA CMBX bid-ask estimates



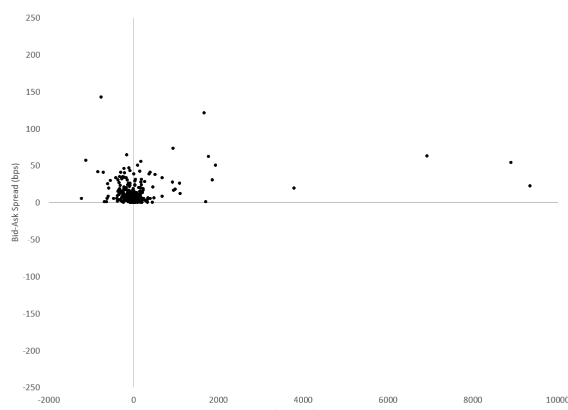
(a) Roll (1984), ‘Preserve sign’



(b) Roll (1984), Restricted (‘zeroed’)



(c) Christopoulos (2020), ‘Absolute Roll’



(d) Thompson & Waller (1987)

This figure depicts the effective bid-ask spread of risk premia (aka. bid-ask spread of ‘Spreads’) for on-the-run AAA CMBX tranche for 2828 daily observations of end of day mark to market values mid market spreads from 11/2007-4/2019. The x-axis for all plots show the autocovariances and the y-axis for all plots shows the effective bid-ask spread of spreads (in bps). Fig. (2a) depicts the effective bid-ask spread for Roll (1984) with the positive autocovariance preserved resulting in negative bid-ask spreads. Fig. (2b). shows the effective bid-ask spread for Roll (1984) with the restriction suggested in the prior literature of zeroing or omitting observations of positive autocovariance (we zero those observations for emphasis). Fig. (2c) shows the effective bid ask spreads using the Absolute Roll Measure of Christopoulos (2020) depicting strictly non-negative effective bid-ask spreads. Fig. (2d) shows the effective bid-ask spread using the technique of Thompson and Waller (1987). These daily values are used as the dependent variables in the OLS and VAR models described in Eq. (25) and Eq. (26), respectively.

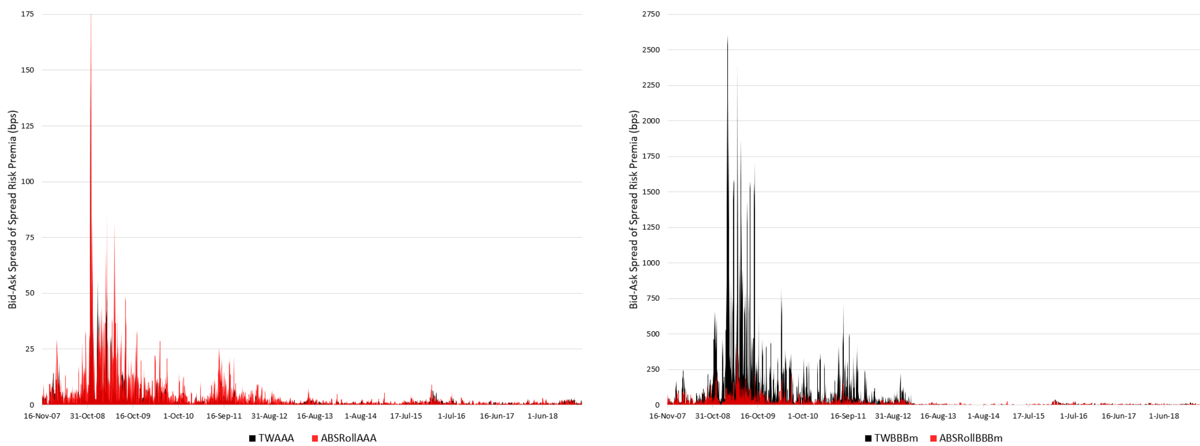
Consistent with the earlier findings of Roll (1984), Harris (1990) and Hasbrouck (2009) we too find positive autocovariance to be frequently observable in our study. In our sample of 2828 observations for AAA and BBB CMBX credit tranches: 1164 (41%) of the AAA observations and 1076 (38%) of the BBB observations exhibited positive autocovariance. To give some context for the implementation of the models, we depict their implementation for the AAA CMBX in Figure 2. Figure 2a shows an example of the application of Model 1 to AAA CMBX risk premia above the risk-free rate with splits in application according to domains described in Eq. (2). The x-axis captures the range of values for the autocovariance while the y-axis captures the range of values for s. There we indeed observe negative bid-ask spreads corresponding to positive autocovariances. Model 2, drops observations that exhibit positive autocovariance in the statistical analysis as suggested by Hasbrouck (2009) and Foucault, Pagano and Roell (2013).

Figure 2b shows an implementation of Model 2 for the same series for AAA CMBX. Dropped values are 'zeroed' for visual emphasis to the right of the origin in the graph. This implementation eliminates about 41% of the sample in the statistical analysis. Figure 2c shows the implementation of Model 3 in Eq. (5). We see a strictly positive bid-ask spread for all CMBX AAA observations, as expected. The positive autocovariance values to the right of the origin on the x-axis in Figure 2c exhibit positive bid-ask spreads on the y-axis in contrast to Figure 2a and Figure 2b. Figure 2d shows the bid-ask spreads for AAA for Thompson and Waller (1987) (Model 4) utilizing the autocovariance from the prior three models as a characteristic, even though it is not included in this estimation. As is evident, the Absolute Roll Measure and the Thompson-Waller methods yield very different results at the same observation date. This is interesting. Next we show the time series for Model 3 and Model 4 which guarantee a non-negative bid-ask spread for each observation date.

Figure 3 shows on-the-run AAA and BBB bid-ask spreads for Model 3 and Model 4 over the sample period. Figure 3a depicts the modelled bid-ask estimates for AAA while Figure 3b exhibits the estimates for BBB.As expected, the AAA bid-ask spreads are categorically narrower than the BBB bid-ask spreads. This is due to the fact that virtually all non-agency CMBS are structured in senior subordinate sequential pay capital structures. As such, losses from defaults are deducted from the bottom of the capital structure with the lowest credit rated tranches being impacted first (Unrated,...,BBB,...,AAA). Since BBB securities will be impacted by losses from defaults before AAA, credit speculators betting against the sector using CMBX will opt to 'buy protection' by paying a fixed premium and receiving floating. As credit spreads widen, the short position will increase in value on a mark-to-market basis. Thus, even if defaults don't materialize, investor expectations of defaults will cause the short leg of lower rated tranches to increase in value more rapidly than higher rated tranches when credit spreads widen.[11]

[11]See An, Deng, Nichols and Sanders (2015), Riddiough and Zhu (2016), Christopoulos (2017) and Real

Figure 3: CMBX bid-ask spreads



(a) AAA bid-ask spreads

(b) BBB bid-ask spreads

This figure depicts the daily time series of effective bid-ask spread of risk premia (aka. bid-ask spread of 'Spreads') for on-the-run AAA CMBX tranche for 2828 daily observations of end of day mark to market values mid market spreads from 11/2007-4/2019. The on-the-run AAA CMBX and BBB/BBB- effective bid-ask spreads are depicted using the on-the-run selection method and modeled using the Absolute Roll Measure of Christopoulos (2020) and Thompson and Waller (1987) methods. The x-axis for all plots show the date and the y-axis for all plots shows the effective bid-ask spread of spreads (in bps). Fig. (3a) depicts AAA bid-ask spreads while Fig. (3b) depicts BBB/BBB- bid-ask spreads.

3.6.3 Statistical summary of the Daily Model

We construct a number of new measures in this paper which are summarized in Table 12.To address the regime shifts in data surrounding the halting of CMBX series issuance from 5/2008 - 1/2013, we provide statistical summary the daily public data and the Daily Model output in Table 12 in three panels. Panel A (11/1/2007-4/17/2019) captures the entire sample. Panel B captures the beginning of the sample to the end of the temporary halt in CMBX issuance (11/1/2007-1/24/2013). Panel C captures the period 2/4/2013-4/17/2019. We use this daily data of observed values and on-the-run values in our first testing of microstructure and risk partition liquidity measures.

As expected, BBB spreads are larger (wider) than AAA spreads which are smaller (tighter). This is due to additional risk premia required by investors in subordinate BBB securities who are more exposed to loss than senior AAA counterparts.

Deal (2020), among others.

Table 12: Daily Data, Bid-Ask, Risk Partition Models

Panel A: 11/16/07-4/2/19	mean	median	min	max	variance	stdev	n
VIX	19.74	17.00	9.14	80.86	91.15	9.55	2822
10yr Tsy	2.64	2.56	1.37	4.30	0.46	0.68	2822
2yr Tsy	1.02	0.75	0.16	3.33	0.62	0.78	2822
Tsy Slope (10s-2s)	161.90	163.00	11.00	291.00	5278.17	72.65	2822
Corp Baa	284.43	274.00	156.00	616.00	6335.29	79.59	2822
Corp Aaa	171.01	174.00	84.00	300.00	1211.61	34.81	2822
Credit Slope (Baa-Aaa)	113.42	100.00	53.00	350.00	2755.99	52.50	2822
CMBX Aaa	158.38	106.95	44.45	847.50	17164.78	131.01	2822
CMBX Baa	2172.00	561.42	290.95	6924.08	4398624.78	2097.29	2822
CMBX CrdSlope	2013.62	458.34	214.28	6314.59	4047916.97	2011.94	2822
Bid/Ask TWBaa	88.63	6.48	0.00	3161.76	63301.09	251.60	2822
Bid/Ask ABSRollBaa	22.93	8.44	0.05	507.96	1504.61	38.79	2822
Bid/Ask TWAaa	2.56	0.87	0.00	142.83	44.27	6.65	2822
Bid/Ask ABSRollAaa	5.20	1.70	0.03	193.37	116.00	10.77	2822
CJdef_pct	0.13	0.13	0.00	0.87	0.00	0.07	2822
CJrates_pct	0.32	0.26	0.00	0.87	0.03	0.17	2822
CJreglq_pct	0.26	0.25	0.00	0.97	0.01	0.11	2822
CJxslq_pct	0.30	0.33	0.00	0.96	0.04	0.21	2822
Panel B: 11/16/07-1/24/13	**mean**	**median**	**min**	**max**	**variance**	**stdev**	**n**
VIX	25.52	22.44	12.43	80.86	118.41	10.88	1296
10yr Tsy	2.96	3.17	1.43	4.30	0.63	0.79	1296
2yr Tsy	0.92	0.75	0.16	3.33	0.56	0.75	1296
Tsy Slope (10s-2s)	203.63	198.00	82.00	291.00	2819.02	53.09	1296
Corp Baa	330.28	306.00	226.00	616.00	7698.42	87.74	1296
Corp Aaa	188.12	181.00	131.00	300.00	884.08	29.73	1296
Credit Slope (Baa-Aaa)	142.16	125.00	74.00	350.00	3917.08	62.59	1296
CMBX Aaa	242.99	191.73	57.63	847.50	23591.11	153.59	1296
CMBX Baa	4226.50	4344.64	982.75	6924.08	1761615.77	1327.26	1296
CMBX CrdSlope	3983.51	4121.04	924.88	6314.59	1628953.33	1276.30	1296
Bid/Ask TWBaa	188.67	70.15	0.00	3161.76	119352.96	345.47	1296
Bid/Ask ABSRollBaa	43.63	27.93	0.74	507.96	2462.15	49.62	1296
Bid/Ask TWAaa	4.69	1.99	0.00	142.83	87.38	9.35	1296
Bid/Ask ABSRollAaa	9.94	5.41	0.07	193.37	209.99	14.49	1296
CJdef_pct	0.13	0.12	0.00	0.87	0.01	0.09	1296
CJrates_pct	0.43	0.45	0.00	0.87	0.03	0.16	1296
CJreglq_pct	0.28	0.29	0.00	0.97	0.01	0.11	1296
CJxslq_pct	0.16	0.09	0.00	0.95	0.03	0.18	1296
Panel C: 2/6/13-4/2/19	**mean**	**median**	**min**	**max**	**variance**	**stdev**	**n**
VIX	14.84	13.81	9.14	40.74	15.68	3.96	1526
10yr Tsy	2.37	2.37	1.37	3.24	0.17	0.41	1526
2yr Tsy	1.10	0.75	0.20	2.98	0.64	0.80	1526
Tsy Slope (10s-2s)	126.45	123.00	11.00	266.00	4632.19	68.06	1526
Corp Baa	245.50	241.50	156.00	363.00	1878.91	43.35	1526
Corp Aaa	156.49	162.00	84.00	226.00	1030.76	32.11	1526
Credit Slope (Baa-Aaa)	89.01	87.00	53.00	154.00	473.71	21.76	1526
CMBX Aaa	86.53	87.85	44.45	171.49	468.80	21.65	1526
CMBX Baa	427.15	420.79	290.95	817.99	7187.70	84.78	1526
CMBX CrdSlope	340.63	344.45	214.28	646.50	6205.90	78.78	1526
Bid/Ask TWBaa	3.68	2.46	0.00	53.78	18.60	4.31	1526
Bid/Ask ABSRollBaa	5.35	4.23	0.05	39.50	18.93	4.35	1526
Bid/Ask TWAaa	0.75	0.56	0.00	6.92	0.55	0.74	1526
Bid/Ask ABSRollAaa	1.18	0.93	0.03	10.23	1.00	1.00	1526
CJdef_pct	0.13	0.13	0.00	0.82	0.00	0.04	1526
CJrates_pct	0.22	0.22	0.00	0.82	0.01	0.09	1526
CJreglq_pct	0.23	0.23	0.00	0.94	0.01	0.10	1526
CJxslq_pct	0.42	0.41	0.00	0.96	0.02	0.14	1526

This table summarizes the daily data used in the paper. Panel A summarizes for the entire sample period 11/2007-4/2019. Panel B summarizes from 11/2007-1/2013. Panel C summarizes from 2/2013-4/2019. The split account for the temporary halt for 5 years of CMBX issuance from 2008 to 2013. The VIX is the CBOE volatility index. Next, 10 year and 2 year Treasury yields are captured as well as their difference in the Treasury Slope (10s-2s). Then the Corporate Bond Spreads (in bps) of Baa and AAA ratings are captured along with their difference, the Corporate Bond Credit Slope (Baa-Aaa). Then aggregations for on the CMBX Spreads (in bps) based on the selection criteria in Eq. (8) are captured for Aaa and Baa credits as well as their differences the CMBX Credit Slope (Baa-Aaa). These are followed by the effective bid-ask spreads of Spreads (in bps) for Baa CMBX using Model 4 Thompson Waller (Bid/Ask TWBaa) and then Model 3 Absolute Roll (Bid/Ask ABSRollBaa), with the corresponding values for Aaa CMBX immediately following them. Finally, the estimated proportions of risk partitions using the Daily Model for the on-the-run CMBX sector index weighted across all CMBX credit ratings in Eq. (10) is provided for default (CJdef_pct), interest rates (CJrates_pct), liquidity (CJreglq_pct), and excess liquidity (CJxslq_pct). The columns to the right of the label report mean, median, min, max, variance, standard deviation (stdev) and the number of observations.

Additionally, there may also be embedded partitions of the risk premia associated with risks other than default for all CMBX including rate risk, liquidity availability and excess liquidity availability as discussed previously. The absolute value of the ratio of the mean to the standard deviation also suggests that AAA securities (1.21) are less volatile than BBB securities (0.96) after normalizing for the mean.

3.6.4 Time series OLS for the Daily Model

For $g \in \begin{cases} 1 & = \text{Roll (1984)} \\ 2 & = \text{Restricted Roll} \\ 3 & = \text{Absolute Roll} \\ 4 & = \text{Thompson and Waller (1987)} \end{cases}$ microstructure models and $k \in \begin{cases} 1 & = \text{Aaa} \\ 2 & = \text{Baa} \end{cases}$ rating, let s_{gkt},represent the corresponding bid-ask spread determined at time t. We examine the statistical relationship between the dependent variables of bid-ask spreads, s_{gkt}, for CMBX, key benchmark market indicators, and liquidity and excess liquidity risk partitions using OLS. We verify the stationarity of all variables using the Dickey-Fuller unit root test and both the liquidity and the excess liquidity indices were found to be stationary. For the remaining independent variables, we took first differences and also confirmed their stationarity with Dickey-Fuller. We consider all models over 2828 daily observations in the sample period 1/2007-4/2019.[12] We regress model bid-ask spreads against the VIX volatility index (VIX), the Treasury Slope (10 year US Treasury yield to maturity - 2 year US Treasury yield to maturity, 'TsySlp'), the corporate credit slope (BBB-AAA, 'CrdSlp'), and each of the liquidity availability (LQ) and excess liquidity availability (XSLQ) indices for the CMBX sector.

The estimates of the ordinary least squares (OLS) regression are determined with

$$\begin{aligned} s_{gkt} = \quad & \alpha + \beta_1 \Delta \mathrm{VIX}_t + \beta_2 \Delta \mathrm{TsySlp}_t + \beta_3 \mathrm{CrdSlp}_t \\ & + \beta_4 \mathrm{LQ}_t + \beta_5 \mathrm{XSLQ}_t + \epsilon_t \end{aligned} \tag{25}$$

The results of the regressions are summarized in Table 13 grouped by credit rating (AAA and BBB). Across *both* credit rating categories Model 1, with preserved autocovariance (negative bid-ask spreads) as described in Eq. (2), is statistically insignificant, while the three other models are statistically significant as measured by the F-test.

[12] We eliminate 6 dates from 1/29/2013 thru 2/5/2013 corresponding to the 're-start' of the CMBX market with the issuance of CMBX Series 6.

Table 13: Time series OLS, AAA

	Roll (1)	Restricted Roll (2)	ABSRoll (3)	TW (4)
Intercept	-0.36589	17.97386***	19.85028***	8.04411***
	0.95751	0.73358	0.76557	0.50276
chgVIX	-0.1203	-0.2673**	-0.16294 .	-0.07544
	0.11786	0.08985	0.09423	0.06188
chgTsySlp	0.09272	-0.01474	-0.11751**	-0.02223
	0.05491	0.04257	0.0439	0.02883
chgCrdSlp	-0.08124	0.19366**	0.28591***	0.08836 .
	0.09726	0.07077	0.07776	0.05107
Lq	2.40796	-21.30327***	-24.37286***	-7.33146***
	2.55098	1.94928	2.0396	1.33945
XSLq	0.19045	-25.47087***	-28.0375***	-12.05928***
	1.301	0.97257	1.04019	0.68312
df	2822	1658	2822	2822
F-test p-val	0.2703	0.00	0.00	0.00
Adj-Rsq	0.0004919	0.3017	0.2114	0.1075

This table summarizes the results from the AAA OLS regression $s_{gkt} = \alpha + \beta_1 \Delta\text{VIX}_t + \beta_2 \Delta\text{TsySlp}_t + \beta_3 \text{CrdSlp}_t + \beta_4 \text{LQ}_t + \beta_5 \text{XSLQ}_t + \epsilon_t$ as defined in Eq. (25). The dependent variable considered are the AAA effective bid-ask spreads, s_{gkt} determined using the four microstructure models of Roll (1984) with positive autocovariance preserved for observations, Roll (1984) augmented with observations of positive autocovariance eliminated, Christopoulos (2020), and Thompson and Waller (1987). The five independent variables are the change in the VIX volatility index ('chgVIX'), the change in the Treasury slope ('chgTsySlope') of 10year - 2year yields, the change in the corporate credit slope ('chgCrdSlp') of BBB-AAA risk premia, the estimated CMBX sector wide indices weighted across all ratings and tranches for liquidity ('Lq') and excess liquidity ('XSLq') as depicted in Fig. (1). The significance codes of '***', '**', '*', and '.' indicate statistical significance at the 0.001, 0.01, 0.05 and 0.1 levels, respectively.

Table 14: Time series OLS, BBB

	Roll (1)	Restricted Roll (2)	ABSRoll (3)	TW (4)
Intercept	8.2623	399.7197***	68.1094***	388.1854***
	7.3778	23.1303	6.9989	18.3606
chgVIX	0.4042	1.5045	-0.1384	0.158
	0.9081	2.8203	0.8615	2.2599
chgTsySlp	0.5833	0.3842	0.2917	0.3894
	0.4231	1.2611	0.4014	1.0529
chgCrdSlp	0.5793	-0.8997	0.65	-1.8348
	0.7494	2.2132	0.7109	1.865
Lq	2.3037	-536.5333***	-63.1772***	-484.008***
	19.6558	61.9319	18.6463	48.9159
XSLq	-3.866	-592.9525***	-86.4366***	-587.0661***
	10.0244	30.6485	9.5096	24.9471
df	2822	1746	2822	2822
F-test p-val	0.7238	0.00	0.00	0..00
Adj-Rsq	-0.0007627	0.1801	0.02843	0.1676

This table summarizes the results from the BBB OLS regression $s_{gkt} = \alpha + \beta_1 \Delta\text{VIX}_t + \beta_2 \Delta\text{TsySlp}_t + \beta_3 \text{CrdSlp}_t + \beta_4 \text{LQ}_t + \beta_5 \text{XSLQ}_t + \epsilon_t$ as defined in Eq. (25). The dependent variable considered are the BBB effective bid-ask spreads, s_{gkt} determined using the four microstructure models of Roll (1984) with positive autocovariance preserved for observations, Roll (1984) augmented with observations of positive autocovariance eliminated, Christopoulos (2020), and Thompson and Waller (1987). The five independent variables are the change in the VIX volatility index ('chgVIX'), the change in the Treasury slope ('chgTsySlope') of 10year - 2year yields, the change in the corporate credit slope ('chgCrdSlp') of BBB-AAA risk premia, the estimated CMBX sector wide indices weighted across all ratings and tranches for liquidity ('Lq') and excess liquidity ('XSLq') as depicted in Fig. (1). The significance codes of '***', '**', '*', and '.' indicate statistical significance at the 0.001, 0.01, 0.05 and 0.1 levels, respectively.

AAA For AAA bid-ask spreads as dependent variable, wider bid-ask spreads correspond to lower volatility as measured by the VIX. This would suggest a lagging response in market pricing compared with observed volatility. This conjectured lagging relationship corresponds to intuition for the significant Models 2, 3 and 4. As bid-ask spreads widen (greater uncertainty) changes in the Treasury slope become more muted (negative) reflecting a general flight to quality. Simultaneously, the changes in the credit slope are increasing (positive), also reflecting market wide concerns over credit. Finally, the signs of both liquidity and excess liquidity indices are negative suggesting that when bid-ask spreads widen, there may be less sector-wide liquidity embedded within market prices. The Adjusted R-squared value of 0.3017 for Model 2[13] exhibits the best explanatory value for the bid-ask estimates, but its scope is limited. In contrast, Model 3 with its Adjusted R-squared value of 0.2114 provides consistently significant estimators for each of the covariates. Model 4 provides less insight.

BBB CMBX In general, the finding for the BBB OLS are worse than the AAA case. The stand-out results are the signs and significance of the liquidity and excess liquidity indices which are negative, and highly significant for Models 2, 3 and 4. The other covariates are insignificant though, in the case of Model 3 the coefficient signs are mostly similar to the AAA case. The exception is in the change of the Treasury slope which exhibits a positive sign. This is encouraging as the credit sensitive portion of the capital structure (BBB) should be less influenced by yield curve dynamics than corresponding senior (AAA) credits.

As in the AAA case, Model 2 model provides the highest explanatory value as measured by the Adjusted R-squared of 0.1801, but it again suffers from the limited sample of 38% less observations than are considered by the other three models. Interestingly, we see lower incidence of positive autocovariance in BBB than AAA. In Model 2 for AAA there are 1164 incidents (2822-1658) while in BBB there are 1082 incidents (2822-1746). This could suggest that investors have more idiosyncratic risk information about loan and bond collateral and are somewhat less apt to simply engage in follow-on pricing exhibited in the more commoditized AAA classes which are protected from default driven losses through subordination. AAA securities are categorically more liquid than BBB which, as subordinate classes, exhibit

[13]Model 2 has 43% less observations than the other models due to dropped values exhibiting positive autocovariance.

categorically wider bid-ask spreads, wider mid-market spreads, and less liquidity and excess liquidity. This is due in part to idiosyncratic concerns over the risk of default on underlying collateral. As such, the regressions for the BBB class of CMBX suggest that other drivers are materially influencing pricing of the securities, which in turn influences the bid-ask spread estimates. It is thus not surprising that the Adjusted R-squared values for the BBB regressions are categorically lower than for AAA. It is interesting that the Adjusted- R-squared value for Model 4 of 0.16786 is higher than the Adjusted R-squared for Model 3 of 0.02843.

These statistics seem to support the distinctions between the senior AAA and the subordinate BBB securities. Interestingly, the behavior of BBB mimics AAA with respect to the pricing of liquidity embedded within market pricing which we investigate further with IRF.

3.6.5 VAR, Granger and IRF for the Daily Model

While the OLS results are good, they are somewhat coarse to the task of liquidity evaluation. OLS relationships are contemporaneous and do not control for autoregressive behavior amongst covariates. In contrast, VAR and IRF techniques do provide such controls and are applied here to CMBX liquidity evaluation.

VAR With VAR, we estimate the 'true' value of the bid-ask spread, s_t, as the dependent variable at time t explained by its own lagged variables, s_{t-n}, and lagged market variables, M_{t-n}. The VAR we implement is of the form

$$s_t = \alpha + \sum_{n=1}^{N} \beta_n s_{t-n} + \sum_{n=1}^{N} \gamma_n M_{t-n} + \epsilon_t \tag{26}$$

The n-period lagged $\sum_{n=1}^{N} \beta_n s_{t-n}$ term serves as the sum of first-differenced bid-ask spread variables while the $\sum_{n=1}^{N} \gamma_n M_{t-n}$ term captures the sum of first differenced market variables. In all cases, the n-period lags are determined by Akaike Information Criteria (AIC), and we permit up to $n = 10$ daily lags (2 weeks of trading). Each market variable has one equation. The current time t observation of each variable depends upon lagged values of bid-ask spread estimates as well as lagged values for the market variable. Ivanov and Kilian (2005) conclude that n-order lag selections are ultimately discretionary to the project. As our lags comport with those found in the earlier literature, we are comfortable with this choice. In the OLS, the liquidity measures categorically exhibit statistical significance and intuitively correct signs, and so we focus on those market variables to generate the IRFs. We restrict the analysis to Model 3 and Model 4 due to their reliability across the sample. The results of the VARs are provided in the Appendix A.3 in Tables 23 and 24. They are as expected with high Adjusted R-squared values ranging from 0.73 to 0.84 and statistical significance for lagged bid-ask dependent variable as independent variable for both models across AAA and BBB credits. Interestingly, lagged values for liquidity indices exhibit statistical significance in Model 3 but less so in Model 4.

Granger As with the VARs, the Granger tests are restricted to Models 3 and 4 and only test relationships between bid-ask spread and liquidity measures. In Panel B, For Models 3 and 4 we find strong statistical evidence at the 0.10% significance level with excess liquidity Granger causing AAA bid-ask spreads. For BBB bid-ask spreads only Model 4 exhibits statistical significance (0.10%) for excess liquidity Granger causing BBB bid-ask spreads, while Model 3 is insignificant. We find weaker evidence of liquidity Granger causing AAA or BBB bid-ask spreads for the exact same relationship across both models in Panel A. Panels C and D exhibit strong statistical support for bid-ask spreads Granger causing liquidity and excess liquidity, which is not surprising given both liquidity and excess liquidity are residual measures. These results are summarized below in Table 15.

Table 15: Granger Causality Test Statistics

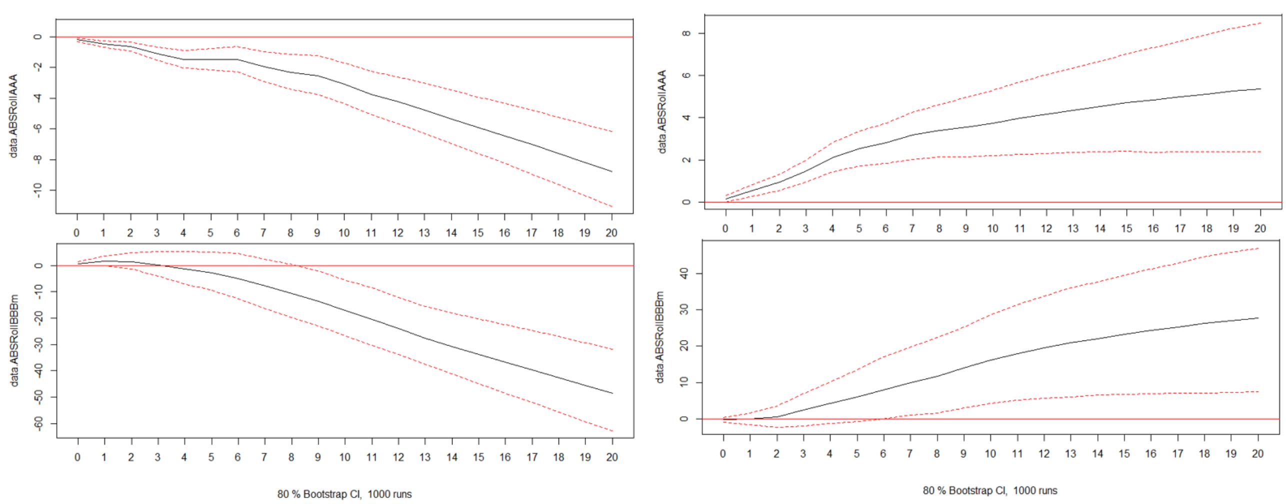
	Panel A: Liquidity~Bid/Ask		Panel B: Excess Liquidity~Bid/Ask	
	AAA	BBB	AAA	BBB
ABSRoll	3.4706	0.6141	17.161	1.7907
	0.01549*	0.6058	4.83e-11***	0.1468
TW	3.1354	2.7673	11.318	12.729
	0.02451*	0.04036*	2.231e-07***	2.914e-08***
				.
	Panel C: Bid/Ask~Liquidity		**Panel D: Bid/Ask~Excess Liquidity**	
	AAA	BBB	AAA	BBB
ABSRoll	5.0702	1.1208	6.9661	7.981
	0.001675**	0.3393	0.0001145***	2.69e-05***
TW	5.449	0.329	7.6033	16.434
	0.0009886***	0.8044	4.614e-05***	1.38e-10***

This table shows the results for the Granger causality tests with F-stat and p-value reported immediately below. All panels show statistics for AAA and BBB cases in separate columns. Rows indicate which effective bid-ask spread model is being use (the Absolute Roll measure of Christopoulos (2020) or Thompson and Waller (1987)). Panel A reports the impulse as CMBX liquidity with the response variables the effective bid-ask spreads. Panel B reports the impulse as CMBX excess liquidity with the response variables the effective bid-ask spreads. Panel C reports the impulse as the effective bid-ask spread with CMBX liquidity as the response variables. Panel D reports the impulse as the effective bid-ask spread with CMBX excess liquidity as the response variables. ***/**/*/’ correspond to 0.1%, 1%, 5% and 10% levels of significance.

IRF IRFs describe a response variable's evolution over a projected time horizon following a one standard deviation exogenous random shock to the initial values of the endogenous variables in VAR using the bootstrapping method. It is a statistical assessment of sensitivity (response) to a one-standard deviation shock (impulse) within the VAR and is conducted as a post-estimation analysis. Following Lütkepohl (2005), a moving average representation (also referred to as a forecast error impulse response, or FEIR, $\Phi_i = \sum_{j=1}^{i} \Phi_{j-1} A_j, i = 1, 2, ...$) is an iterative formulation for Φ_i with $\Phi_0 = I_K$ and A_j the coefficient vector for lag j. To correctly account for the contemporaneous correlation between residuals of the covariates in the VAR, which is not accounted for in the coefficient vectors A_i, we orthogonalize the symmetric positive semi-definite variance covariance matrices, Σ, by determining P (a lower triangular matrix) using the Cholesky decomposition method that satisfies $\Sigma = PP^{-1}$. The corresponding orthogonal IRF, which accounts for contemporaneous correlations amongst regressors, is given by

$$\Theta_i^o = \Phi_i P \tag{27}$$

Figure 4: IRF's from Liquidity and Excess Liquidity Indices onto bid-ask spreads



(a) Excess Liquidity x bid-ask (b) Liquidity x bid-ask

This figure provides the cumulative IRFs for effective bid-ask spreads modelled with the AAA and BBB Absolute Roll Measure of Christopoulos (2020) VARs for liquidity and excess liquidity with lag order 10 periods. The upper and lower dashed lines represent the upper and lower bounds of the 80% confidence interval while the line in the center depicts the values of IRF over time. It is the moving average evolution forecast of the response variable (effective bid-ask spreads) over 20 days (about 1 trading month) years following the shock of CMBX liquidity and CMBX excess liquidity. The x-axis, shows forward days, for each of the variables. The y-axis shows the cumulative percentage change from time $t = 0$ to $t = h$. The top chart of Fig. (4b) depicts the AAA CMBX effective bid-ask spread response to a 1 standard deviation shock in CMBX liquidity, while the bottom chart of Fig. (4b) depicts the BBB CMBX effective bid-ask spread response to a 1 standard deviation shock in CMBX liquidity. The top chart of Fig. (4a) depicts the AAA CMBX effective bid-ask spread response to a 1 standard deviation shock in CMBX excess liquidity, while the bottom chart of Fig. (4a) depicts the BBB CMBX effective bid-ask spread response to a 1 standard deviation shock in CMBX excess liquidity.

When calculating the IRFs we use exogenous one standard deviation shocks to the impulse variables such that for each forward period, h, we are able to determine the forecast results as shown in Figure 4. The plots depict the cumulative percentage change (y-axis) in the bid-ask spread using Model 3 at time $t \in [0, h]$ on the x-axis following exogenous shocks of liquidity and excess liquidity risk partitions. The upper and lower dashed lines represent the upper and lower bounds of the 80% confidence interval while the line in the center represents the cumulative value of the IRF. The interpretation is one of a cumulative moving average forecast of bid-ask spreads over $h \in [0, 20]$ trading days (about 1 month) following the respective shocks. AAA are depicted in the top panel while and BBB are depicted in the bottom panel. The results are good. The confidence interval boundaries lie above the zero partition indicating a clear response to the signal based upon the VAR interrelationships.

The results are good. Figure 4a shows the impact on bid-ask spreads for a one standard deviation shock in excess liquidity. There we see the the cumulative percentage response of bid-ask spreads for AAA CMBX is to tighten by about 9 bps with BBB CMBX tightening by about 45 bps over the forecast period. Figure 4b tells a contrasting story with the cumulative percentage response of bid-ask spreads to a one standard deviation impulse shock to the liquidity index widening for AAA CMBX by about 5 bps following with BBB CMBX widening by about 25 bps.

As liquidity and excess liquidity partitions contemplate portions of market spreads that are not accounted for by rate volatility and default risk, such liquidity partitions can be thought of as additional compensation in market spreads in excess of model articulated risks. Liquidity availability can be subsumed within observed risk premia as noted in Jarrow (2007), Bao, Pan and Wang (2011), Carr and Yu (2012) and Christopoulos (2017), among others. Excess liquidity captures the condition where total theoretical risk premia implied by simulated risk neutral prices *exceeds* the observed risk premia implied by market prices. Increases in excess liquidity instantaneously should correspond with tighter bid-ask spreads. And thus makes sense in the IRF that increases in excess liquidity forecast future bid-ask spread tightening. Excess liquidity is the most extreme form of liquidity assessment and it is plausible that even if excess liquidity is not actively revealed to the market as yet, that other signals correlated with excess liquidity are signalling to market actors, resulting in assessments that the bonds are inexpensive relative to their risks.

The more nuanced case is in revealed liquidity availability. As a theoretical liquidity availability increases and the market is unaware of this, it can suggest that market is simply uncertain relative to the underlying risks. This would be consistent with the findings of Bao, O'Hara and Zhou (2018) where the impact results in more restrictive risk policy not substantiated by underlying risks. If market spreads are widening in excess of the theoretical assessment of default risks, then mechanically liquidity availability will increase. But if no one knows about the partitioning, then all the market sees is market spreads widening. It is thus plausible that the liquidity shock in the IRF will correspond to future uncertainty evidenced by projected bid-ask spread widening, even though we would hope that with enhancements to information content such as this, that in future studies, that would reverse. Finally, we note that the Daily Model is well specified because the four elements of the Daily Model are not collinear. The proof of this is found in Appendix A.4.

As shown in the OLS, liquidity risk partitions play a significant role in CMBX price formation (our *first main result*). They give rise to our time series analyses with VAR, Granger and IRFs. This produces our *second main result* from the IRFs that liquidity partitions also have a significant effect on future observed CMBX liquidity as measured by projected effective bid-ask spreads. This establishes a theoretical relationship for CMBX between microstructure estimates of liquidity in effective bid-ask spreads and estimated reduced form assessments of liquidity. Together these results validate the Daily Model.

4 The Intraday Model (during Covid-19 pandemic)

This section introduces the Intraday Model and uses all indexed investment grade CMBX classes (AAA,AJ/AS, AA, A, BBB, BBB-) in the validation of the risk partitions. Unlike the previous section (and motivation for this paper) there are no observable CMBX intraday prices (spreads). And so it is not possible to validate the risk partitions with observed market prices or, even theoretical effective bid-ask spreads. However, the theoretical underpinning of CMBX risk partitions provide insights into the related REIT sector. We disclose that relationship in this section and use it to validate with ICAPM.

4.1 Instantaneous changes

Since the Daily Model, validated in the previous section, is based on changes in explanatory variables in Eq. (29) there is nothing in principle that prevents us from increasing the frequency of estimation over shorter intervals for all risk components. In this section we increase the frequency of linear estimation from daily to 15 second intervals, *intraday,* for each trading day in the Covid-19 pandemic in our sample (the 'Intraday Model').

For the Intraday Model, all mathematics apply to all trading days u and time t is the intraday time index in 1560 15 second intervals from 9:30:15am to 4:15:00pm EST. Once the daily initial conditions have been determined, for each trading day u intraday changes in risk composition are then modelled as a zero-centered function of the evolution of the factors

$$\Delta y_{jk}(t) = \sum_{i=1}^{5} \eta_{ijk} \Delta f_i(t) + \varepsilon_{jk}^{\Delta}(t) \tag{28}$$

for the j-th risk partition of the k-th bond at time t. We compute a covariance matrix such that η_{ijk} is the covariate of the i-th factor with the corresponding bond's corresponding risk component. The coefficients η_{ijk} are determined through OLS and $\varepsilon_{jk}^{\Delta}(t)$ has expected mean zero with standard deviation $\sqrt{\eta_{0jk}}$. We normalize the sum of the proportions for the intraday risk decomposition to 1 as was done previously for daily observations in Eq. (17).

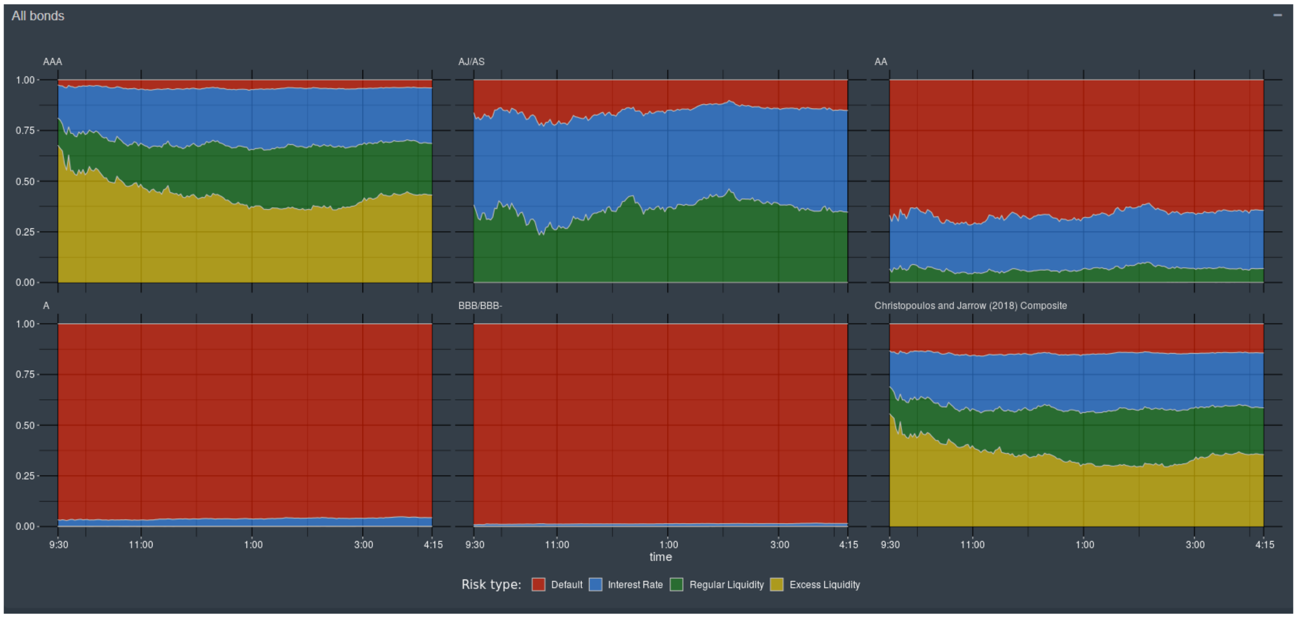
The final risk composition is then calculated using the initial condition and the instantaneous changes:

$$y_{jk}(t) = y_{jk}(t_0) + \sum_{t'=1}^{t} \Delta y_{jk}(t') \tag{29}$$

where the *intraday* estimation of risk components $y_{jk}(t)$ is the j-th component of the risk partition for bond k at time t on trading day u. The first term on the righthand side, $y_{jk}(t_0)$, is the initial value of the risk component intraday and the second term $\sum_{t'=1}^{t} \Delta y_{jk}(t')$ the intraday sum of the changes in the risk component defined in Eq. (28).

Although the time scales for the training data and intraday time steps of 15 seconds are vastly different, they appear to scale well. What we see in Figure 5 is the first-ever intraday evolution of risk decompositions, $y_{jk}(t)$ as defined in Eq. (29) for CMBX over two

Figure 5: CMBX intraday risk decomposition (15 second intervals)



(a) CMBX intraday, 20200417

(b) CMBX intraday, 20201204

This figure provides proportional intraday CMBX risk decomposition in 1560, 15 second intervals. The four components of risk modelled are default, interest rates, liquidity and excess liquidity. The x-axis capture the time intervals from 9:30:15am to 4:15:00pm.EST The y-axis captures the estimated proportion of CMBX risk embedded within CMBS spreads as determined in Eq. (17). Fig. (6a) captures the intraday evolution of CMBX proportional risk decompositions on April 17, 2020 and Fig. (6b) captures the intraday evolution of CMBX proportional risk decompositions on December 4, 2020. Separate evolutions are provided for each of the investment grade CMBX tranches, as well as for the weighted average composite index across all investment grade CMBX as shown monthly in Christopoulos and Jarrow (2018).

trading days for all $j \in [1,4]$ risk partitions and $k \in [1,6]$ credit ratings. The risk partitions are observed for $t \in [1,1560]$ consecutive 15 second intervals in length per trading day. The first estimation occurs at or after 9:30:15 seconds following the market open; the last estimation occurs at or about 4:15:00pm. The variability at the open occurs because the implementation requires the information released at 9:30:00am, the variability at the close is due to reporting delays which may occur slightly before or slightly after 4:15:00pm. The risk partitions at the tranche level are projections based on the indexed training set computed from simulated primitives in Eq. (9). Figure 5 investment grade CMBX bond risk partitions for the investment grade classes and for the aggregate composite ('Christopoulos and Jarrow (2018) Composite').

The risk composites vary considerably across credit rating classes, days, and intraday. The intraday decompositions reflect updated live data for 30 REITs, 10 US Treasuries, and the VIX index as previously described. Figure 6a shows the evolution for $u =$ April 17, 2020 in the early stages of the Covid-19 global pandemic with the Dow Jones Industrial Average (Dow) closing 23537, while Figure 6b shows the evolution of the risk decomposition indices on $u =$ December 4, 2020 with the Dow closing at 30217. The sum of all compositions on all intervals are normalized to 1. Although we show only two trading dates, these evolutions are captured intraday in 15 second intervals for all 240 trading days between April 7, 2020 and April 8, 2021 in this study.[14]

As noted in Christopoulos (2017), model estimated liquidity of CMBX vary considerably across credit ratings and time. AAA CMBS securities make up about 80 percent of the CMBS market due to the senior/subordinate capital structure as noted in An, Deng, Nichols and Sanders (2015) and Riddiough and Zhu (2016) while subordinate BBB- CMBS make up less than 5 percent of the market. As BBB- securities are more immediately exposed to loss manifestation following a default event than AAA securities a smaller proportion of the risk compensation above the risk free rate for those securities should be associated with liquidity availability when compared with AAA. This dynamic is confirmed in Christopoulos (2017) and Christopoulos and Jarrow (2018) in indexed form from 2007-2014 in monthly simulations and is confirmed again in the results of this paper with bid-ask comparisons with such measures daily (as previously discussed), and now as evident, intraday.

[14] These values are available every trading day. See https://risktape.warg.wotnrisk.com/ for more details.

4.2 Validation of the Intraday Model

4.2.1 Statistical summary for the Intraday Model

Table 16: Summary statistics of cumulative changes of intraday CMBX risk partitions

Panel A: Default	AAA	AJ	AA	A	BBB	BBB-
min	0.483416	0.338351	0.494514	0.609670	0.717165	0.823893
max	1.820368	8.403204	6.036482	6.121692	1.913765	1.400509
mean	1.000102	1.000104	1.000046	1.000039	1.000012	1.000005
median	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
variance	0.000212	0.000380	0.000156	0.000147	0.000020	0.000006
stdev	0.014562	0.019491	0.012485	0.012127	0.004425	0.002384
obs	275370	275370	275370	275370	275370	275370
Panel B: Rate risk	**AAA**	**AJ**	**AA**	**A**	**BBB**	**BBB-**
min	0.702365	0.610945	0.530693	0.583500	0.476037	0.599777
max	1.518098	1.742393	2.244835	1.871355	1.976362	1.469975
mean	1.000010	1.000137	1.000058	1.000042	1.000061	1.000024
median	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
variance	0.000019	0.000281	0.000123	0.000086	0.000123	0.000041
stdev	0.004340	0.016768	0.011092	0.009273	0.011089	0.006433
obs	275370	275370	275370	275370	275370	275370

Panel C: Liquidity	AAA	AJ	AA	A	BBB	BBB-
min	0.735970	0.330755	0.030144	0.016780	0.275537	0.134313
max	1.707165	8.456980	9.348418	27.908990	2.411473	6.462325
mean	1.000036	1.000466	1.000559	1.000738	1.000176	1.000397
median	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
variance	0.000078	0.001217	0.001370	0.007370	0.000361	0.000976
stdev	0.008806	0.034890	0.037017	0.085848	0.019004	0.031240
obs	275370	275370	275370	275370	275370	275370
Panel D: XS liquidity	**AAA**	**AJ**	**AA**	**A**	**BBB**	**BBB-**
min	0.346666	0.136986	0.508825	0.508825	0.508825	0.140683
max	3.379393	7.812876	2.413748	2.413748	2.413748	6.325317
mean	1.000147	1.000329	1.000074	1.000074	1.000074	1.000184
median	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
variance	0.000333	0.000868	0.000152	0.000152	0.000152	0.000719
stdev	0.018239	0.029465	0.012328	0.012328	0.012328	0.026816
obs	275370	275370	275370	275370	275370	275370

This table provides summary statistics of intraday CMBX risk partitions. The columns show the investment grade credit rating class and the rows the summary statistics of minimum (min), maximum (max), mean, median, variance, standard deviation (stdev), kurtosis and the number of observation (obs). Panel A summarizes for default risk, Panel B summarizes for interest rate risk, Panel C summarizes for liquidity, while Panel D summarizes for excess (XS) liquidity.

Table 16 provides a statistical summary across all observations intraday for default, rates, liquidity and excess liquidity risk partitions. The summary results generally follow intuition with some new insights and we exhibit all investment grade classes. Focusing first on intraday default risk pricing, we observe AAA CMBX appear to exhibit substantially greater volatility than BBB-, with only the AJ exhibiting greater volatility. Rates volatility pricing intraday is much calmer than default intraday pricing across most credits with the exception of the

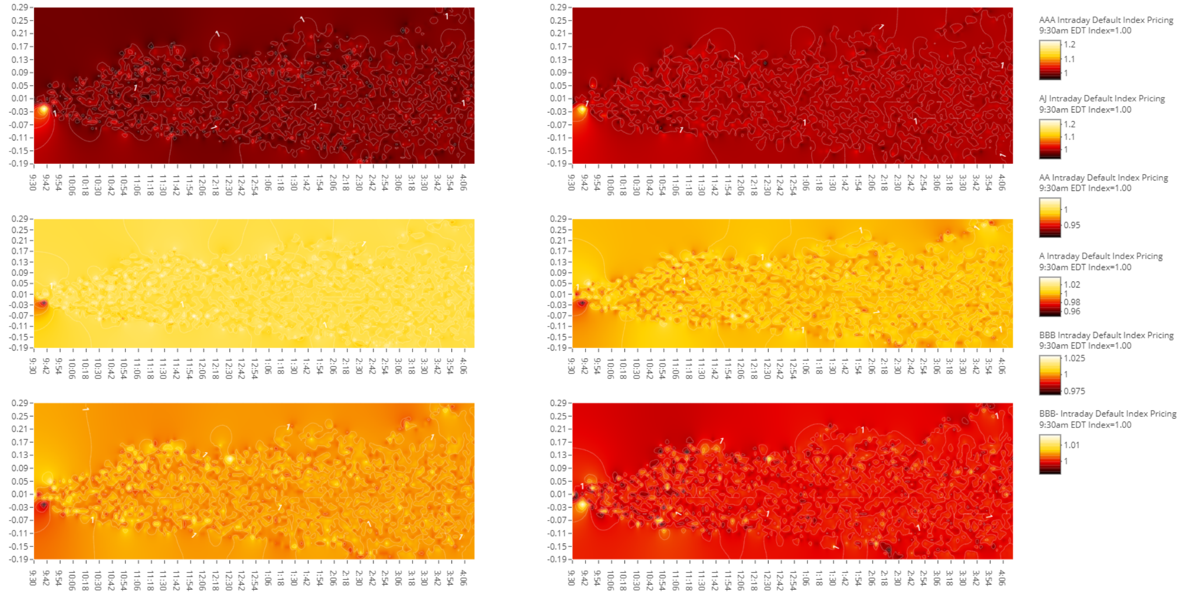
BBB class. Again, as with default, the AJ class appears to exhibit higher volatility than the AAA class. This relatively higher volatility for the AJ class compared with the AAA class repeats across all four risk measures. Finally, as we are concentrating on liquidity, the low nominal amounts of both liquidity and excess liquidity in BBB- classes should result in higher volatility estimates reflective of scarce liquidity availability. Min and max values follow intuition suggesting that the capital structure allocation concentrates mispricing in the mid-section with the endpoints of AAA and BBB- exhibiting more regular behavior.

4.2.2 Cross sectional visualizations for the Intraday Model

While the summary information in Table 16 is interesting from an academic perspective, it is fundamentally limited for two reasons. First, the values estimated intraday still must use an initial set of simulated values and market spreads. The simulated risk decomposition studies of Christopoulos (2017) and Christopoulos and Jarrow (2018), reflecting actual cashflow and updated credit and prepayment profiles, end in 2014.

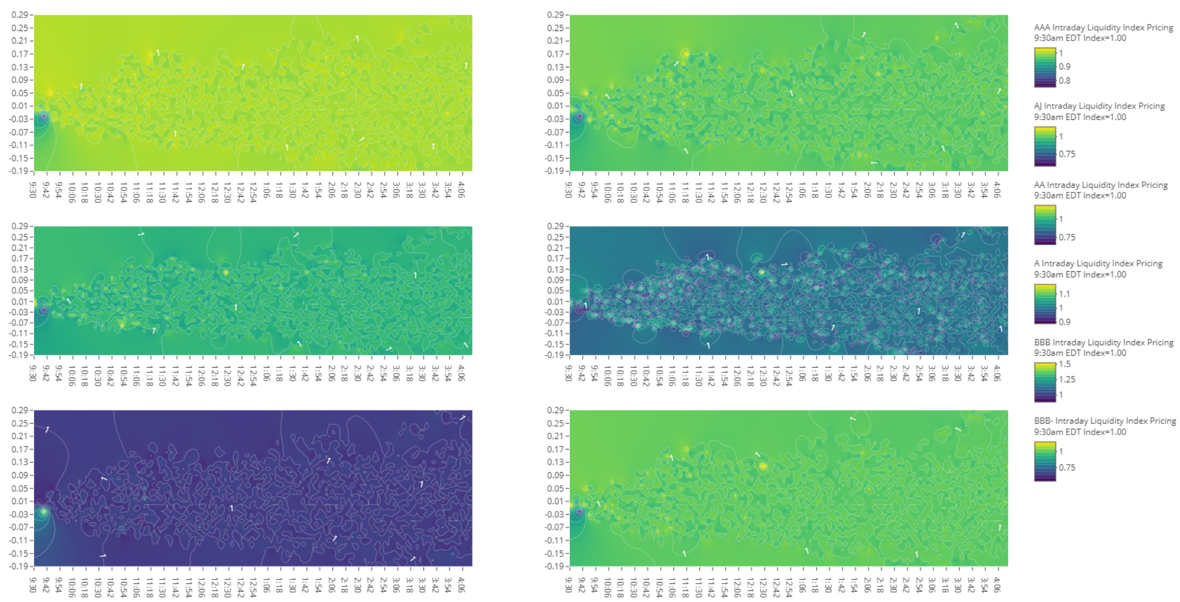
Second, the initialized spread value informing these evolutions are restricted to March 18, 2020 because no additional daily data was made available to us after that date. In this sense, the results depicted are essentially cross-sectional intraday stress tests of the response of risk decompositions holding market spreads as of March 18, 2020 constant. In this sense the limitations of the initialization restrict the validity of the intraday time series values to cross sectional analysis, and not time series analyses. Each day is a valid time series and the collection of all dates representing the set for cross section evaluation. At the same time, because the real time estimations are price independent, observing their changes over time yield some unique insights. One way to approach the evaluation of cross section of daily time series is through the heat map approach using binning. We want to cast the daily risk pricing data against both time and a relevant variable. The VIX provides such a variable as a dominant explanatory variable in the previous daily historical analysis.

Figure 6: Default cross section (all observations)



This figure provides the cross section across all 275370 observations of default risk composition for each of the investment grade CMBX tranches from AAA (top-left) to BBB- (bottom right). The x-axes reflect 3 minute intervals binned from the 15 second interval values while the y-axes capture the log change of the VIX from the start of the trading day (with $t = 0$, 9:30am) until close. The y-axes are partitioned in increments of 0.01. The z-axes are the heat maps for default risk with non-constant upper and lower boundaries, but identical colorscale palettes. The contour lines and hue of indicate higher or lower cumulative changes in default risk across all days in the sample period at identical times.

Figure 7: Liquidity cross section (all observations)



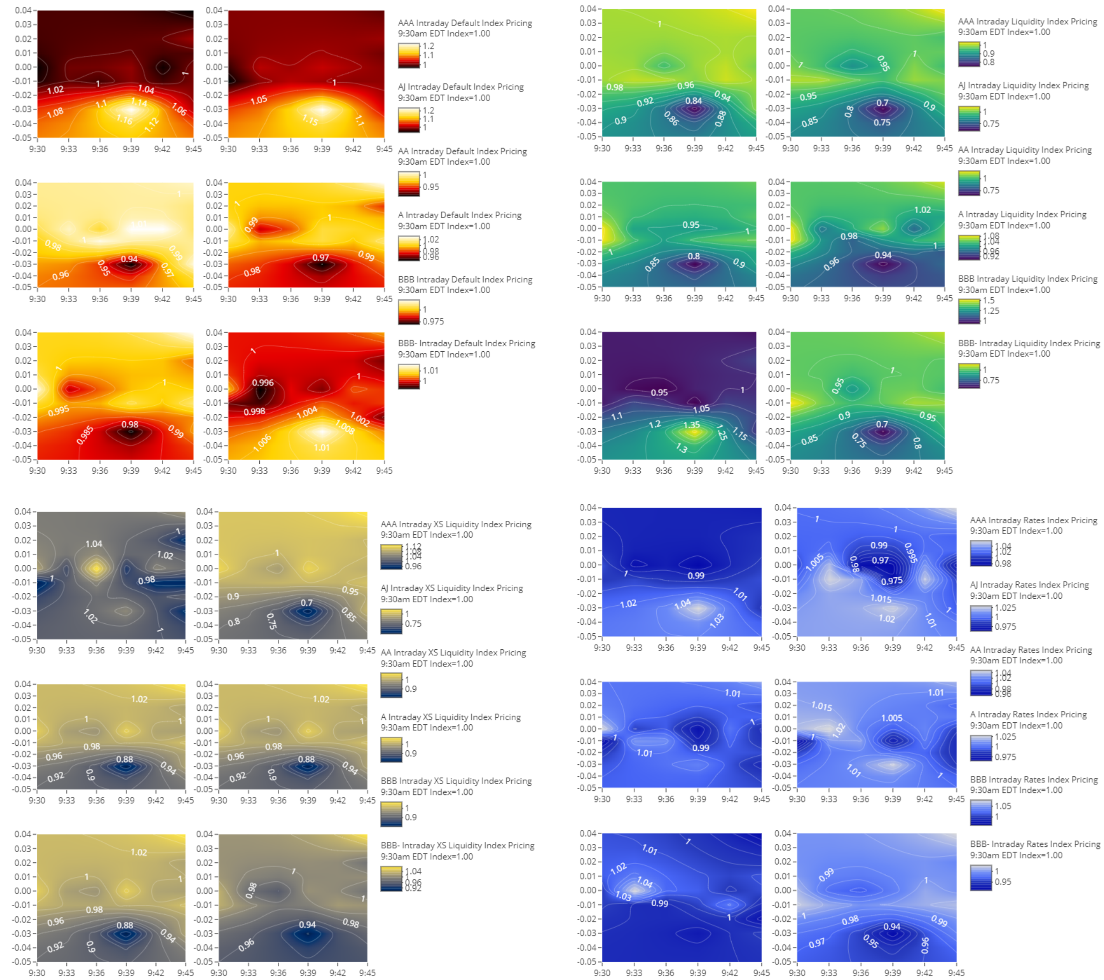
This figure provides the cross section across all 275370 observations of liquidity risk composition for each of the investment grade CMBX tranches from AAA (top-left) to BBB- (bottom right). The x-axes reflect 3 minute intervals binned from the 15 second interval values while the y-axes capture the log change of the VIX from the start of the trading day (with $t = 0$, 9:30am) until close. The y-axes are partitioned in increments of 0.01. The z-axes are the heat maps for liquidity risk with non-constant upper and lower boundaries, but identical colorscale palettes. The contour lines and hue of indicate higher or lower cumulative changes in liquidity risk across all days in the sample period at identical times.

Figure 6 shows 275370 observations computed for each of the intraday default risk partition pricings. We use a double binning method to observe. The x-axes reflect 3 minute intervals binned from the 15 second interval values while the y-axes capture the log change of the VIX from the start of the trading day (with $t = 0$, 9:30am) until close. The y-axes are partitioned in increments of 0.01. The z-axes are the heat maps with non-constant upper and lower boundaries, but identical colorscale palettes. This is purposeful, as each of the classes may have inherently different sensitivities to market indicators. The plots represent the data for each of the CMBX classes with AAA in the upper left, AJ upper right, and so forth, with BBB- in the bottom right. The color saturation produced for each plot is interesting during the Covid period. Together they suggest somewhat lower intraday default risk pricing for AAA, AJ and BBB- classes compared with the AA, A and BBB classes. In Figure 7 we take the same vantage point but for liquidity availability. Here, we see somewhat greater intraday liquidity availability for AAA with the least liquidity available for A and BBB classes as indicated by the color palettes and contour lines.

These perspectives document some important facts in theoretical observation of CMBX risk pricing during the Covid period, one of the most interesting being an apparent regular 'spot' of volatility in the first 15 minutes of trading in the cross section. This repeats across all instruments and risk partitions in our sample. Figure 8 zooms in on this interval for each of the risk partitions for all classes to revealing more detail of the phenomenon. There we see 'peaks' as indicated by the contour lines exceeding 1, and 'valleys' as indicated by contour lines less than one clustered about 9:39am. The subplots depict risk pricing for default, liquidity, excess liquidity and rates indices for all investment grade credit rating classes.

For example, consider in Figure 8 the case of intraday liquidity availability pricing in the second composite of six plots on the top right. At about 9:39am we observe considerable cross-sectional deterioration in liquidity availability for AAA, AJ, AA and BBB- classes with contour labels in 'valleys' ranging from 0.70 to 0.84. In contrast class A class shows relatively more modest deterioration in liquidity availability at about 0.94 while the BBB class shows expansion of liquidity availability of about 1.35. These types of comparisons can be made for all credit ratings classes and all estimated risk partitions generated by the Intraday Model.

Figure 8: All partitions, all dates, 9:30:00am to 9:45:00am



This figure provides the cross section across all 275370 observations of risk composition for each of the four risk components (default, top left), liquidity (top right), excess (XS) liquidity (bottom left) and interest rates (bottom right). Each of the four risk component contain six charts depicting the investment grade CMBX tranches from AAA (top-left) to BBB- (bottom right). The x-axes reflect 3 minute intervals binned from the 15 second interval values from 9:30:15am to 9:45:00am EST. The y-axes capture the log change of the VIX from the start of the trading day (with $t = 0$, 9:30:15am) until 9:45:00am. The y-axes are partitioned in increments of 0.01. The z-axes are the heat maps for the risk components with non-constant upper and lower boundaries, but identical colorscale palettes. The contour lines and hue of indicate higher or lower cumulative changes in liquidity risk across all days in the sample period at identical times.

Table 17: Open Summary during Covid

variable	mean	median	min	max	variance	stdev	n
VIX	26.7659	25.6000	16.5500	47.3400	31.88860	5.6470	246
IN_DRE	37.9874	38.5400	30.4000	43.0600	6.83532	2.6144	246
IN_FR	41.0038	41.5825	33.1700	47.3700	8.22299	2.8676	246
IN_PLD	98.7801	99.5750	81.4700	109.7950	35.65076	5.9708	246
IN_SELF	4.0519	4.0000	3.5200	5.0500	0.08360	0.2891	246
LO_HST	12.8944	11.8875	9.1400	18.4200	4.94857	2.2245	246
LO_MAR	109.4389	100.5550	72.7600	155.7419	495.52655	22.2604	246
LO_WYND	37.0577	33.4700	21.0900	51.7500	93.63226	9.6764	246
LO_MGM	24.6343	22.3325	12.1100	41.7300	63.82471	7.9890	246
MF_AVB	162.0976	159.2000	133.1000	193.5300	145.65429	12.0687	246
MF_ELS	62.7243	62.7025	55.8600	68.2900	5.18841	2.2778	246
MF_EQR	60.1919	59.3700	46.0100	75.9600	37.04669	6.0866	246
MF_UDR	37.7456	37.4550	29.9100	45.6700	11.21798	3.3493	246
OF_BXP	91.4597	91.0200	70.7300	108.5900	74.19748	8.6138	246
OF_CLI	14.0493	13.8825	10.4000	18.5100	2.23470	1.4949	246
OF_HIW	37.9485	38.0200	29.4500	45.1700	10.21290	3.1958	246
OF_SLG	54.5778	50.6400	36.8300	77.1200	93.46684	9.6678	246
OF_VNO	38.3335	37.5750	29.9550	49.0300	16.31488	4.0392	246
OT_BKD	3.8272	3.3900	2.4400	6.7500	1.36962	1.1703	246
OT_NNN	37.3254	36.8075	27.1050	45.8552	17.25113	4.1534	246
OT_PSB	133.0409	131.7250	110.3200	159.8700	106.10071	10.3005	246
OT_WPC	67.5772	68.2300	54.5500	75.0000	12.15876	3.4869	246
RT_KIM	13.6444	12.6550	8.3400	19.7200	8.91863	2.9864	246
RT_REG	45.0055	44.4750	34.0300	59.8800	38.80736	6.2296	246
RT_SPG	78.3857	69.6225	49.7200	120.4975	364.88435	19.1019	246
RT_TCO	40.0019	40.4050	28.1450	46.8000	13.19712	3.6328	246
TSY_3MO	0.0854	0.0880	0.0030	0.2350	0.00149	0.0386	246
TSY_5YR	0.4095	0.3600	0.2060	0.9710	0.02983	0.1727	246
TSY_10YR	0.8936	0.7735	0.5260	1.7490	0.10170	0.3189	246
TSY_30YR	1.6290	1.5460	1.1380	2.4960	0.11141	0.3338	246
AAA_def	1.0131	1.0015	0.6218	1.8204	0.01493	0.1222	246
AAA_rates	1.0015	1.0001	0.8004	1.5181	0.00313	0.0560	246
AAA_reglq	1.0046	0.9995	0.7360	1.3345	0.00616	0.0785	246
AAA_xslq	1.0118	0.9994	0.3467	3.1465	0.04744	0.2178	246
AJ_def	1.0457	1.0008	0.5545	8.4032	0.23584	0.4856	246
AJ_rates	1.0023	0.9998	0.8048	1.3263	0.00365	0.0604	246
AJ_reglq	1.0303	0.9959	0.3308	2.8615	0.10486	0.3238	246
AJ_xslq	1.0308	0.9975	0.1370	4.5724	0.11270	0.3357	246
AA_def	1.0277	1.0000	0.5937	6.0365	0.10726	0.3275	246
AA_rates	1.0082	1.0003	0.6064	2.2448	0.01458	0.1207	246
AA_reglq	1.0092	0.9982	0.0301	3.0219	0.07331	0.2708	246
AA_xslq	1.0087	0.9992	0.5088	2.4137	0.02058	0.1435	246
A_def	1.0219	0.9997	0.6108	6.1217	0.11167	0.3342	246
A_rates	1.0084	0.9995	0.7212	1.8714	0.00939	0.0969	246
A_reglq	1.0013	1.0004	0.0168	3.4114	0.05614	0.2369	246
A_xslq	1.0087	0.9992	0.5088	2.4137	0.02058	0.1435	246
BBB_def	1.0050	0.9999	0.7172	1.9138	0.00514	0.0717	246
BBB_rates	1.0101	1.0001	0.6726	1.4887	0.00836	0.0914	246
BBB_reglq	0.9934	0.9982	0.3555	1.9290	0.02679	0.1637	246
BBB_xslq	1.0087	0.9992	0.5088	2.4137	0.02058	0.1435	246
BBBm_def	1.0029	1.0001	0.9184	1.4005	0.00095	0.0308	246
BBBm_rates	1.0012	1.0000	0.5998	1.3887	0.00533	0.0730	246
BBBm_reglq	1.0430	0.9956	0.1343	5.0141	0.15543	0.3942	246
BBBm_xslq	1.0062	0.9992	0.3763	2.7905	0.03548	0.1884	246

This table summarizes the observations of intraday data at the open of the trading day 9:30:15ET for 246 trading days during the pandemic (4/2020 - 4/2021). The first column provides the abbreviation for the variable name. The VIX is the CBOE volatility index. This is followed by the prices of 25 REITs with the name a composite made up of the property type (industrial (IN), hotel/lodging (LO), multifamily (MF), mixed use/other (OT), office (OF) and retail (RT)) and the REIT's stock market ticker. Following the REITs are US Treasury yields with ticker representing the 4 maturities of 3 month, 5 year, 10 year and 30 year. The remaining values are the simulated risk partitions indexed in Christopoulos and Jarrow (2018) in cumulative change form. The ticker is a composite of the 4 types of risk partitions default (def), interest rates (rates), liquidity (reglq) and excess liquidity (xslq) combined with the credit rating class names of AAA, AJ, AA, A, BBB and BBB- (BBBm). Each of the columns to the right of the ticker report statistics across all 246 observations.

Table 18: Close Summary during Covid

variable	mean	median	min	max	variance	stdev	n
VIX	26.6096	25.6550	16.9900	45.4300	29.64890	5.4451	246
IN_DRE	38.0191	38.4550	30.4000	43.0300	6.75872	2.5998	246
IN_FR	41.0291	41.6200	33.3500	47.4500	8.21028	2.8654	246
IN_PLD	98.7703	99.6100	82.8500	109.5300	36.53448	6.0444	246
IN_SELF	4.0505	4.0000	3.5622	5.0500	0.08343	0.2888	246
LO_HST	12.8848	11.9275	9.4500	18.4200	4.91837	2.2177	246
LO_MAR	109.2835	100.3450	75.2600	157.5000	492.77623	22.1986	246
LO_WYND	37.0491	33.2800	21.3900	51.7500	92.64766	9.6254	246
LO_MGM	24.6201	22.0400	12.6800	42.2000	63.62672	7.9766	246
MF_AVB	161.9634	159.0800	132.7800	193.6100	143.79254	11.9914	246
MF_ELS	62.7417	62.6450	56.1500	68.2900	5.27037	2.2957	246
MF_EQR	60.1311	59.3700	46.2300	74.9600	37.29848	6.1072	246
MF_UDR	37.7246	37.5050	29.6000	45.5600	11.21592	3.3490	246
OF_BXP	91.4361	91.2000	71.1500	108.6600	73.53062	8.5750	246
OF_CLI	14.0223	13.8000	10.4100	18.6900	2.24609	1.4987	246
OF_HIW	37.9085	38.0250	29.7050	45.2300	10.27220	3.2050	246
OF_SLG	54.5360	51.1750	36.8200	77.7900	92.16894	9.6005	246
OF_VNO	38.2630	37.4800	30.1900	49.0100	16.01428	4.0018	246
OT_BKD	3.8215	3.4050	2.4350	7.0500	1.37638	1.1732	246
OT_NNN	37.3067	36.7350	27.6000	45.6000	16.87934	4.1084	246
OT_PSB	133.0165	131.8750	110.3100	159.9100	107.41229	10.3640	246
OT_WPC	67.5539	68.1400	54.5600	74.3100	11.90968	3.4510	246
RT_KIM	13.6380	12.7100	8.3000	19.7000	8.91577	2.9859	246
RT_REG	44.9848	44.4500	34.0500	59.6500	38.49700	6.2046	246
RT_SPG	78.2918	69.4800	51.2000	121.1400	363.46241	19.0647	246
RT_TCO	40.0404	40.4700	32.8700	46.6900	12.51065	3.5370	246
TSY_3MO	0.0850	0.0880	0.0030	0.2100	0.00149	0.0386	246
TSY_5YR	0.4094	0.3620	0.1950	0.9410	0.02932	0.1712	246
TSY_10YR	0.8942	0.7630	0.5150	1.7460	0.10130	0.3183	246
TSY_30YR	1.6306	1.5450	1.1620	2.4760	0.11075	0.3328	246
AAA_def	0.9997	0.9999	0.9806	1.0272	0.00002	0.0039	246
AAA_rates	1.0001	1.0000	0.9971	1.0066	0.00000	0.0010	246
AAA_reglq	1.0001	1.0001	0.9745	1.0138	0.00001	0.0032	246
AAA_xslq	1.0006	1.0000	0.9696	1.0326	0.00004	0.0060	246
AJ_def	1.0001	1.0000	0.9913	1.0217	0.00001	0.0026	246
AJ_rates	1.0002	1.0001	0.9666	1.0247	0.00002	0.0040	246
AJ_reglq	1.0001	1.0000	0.9176	1.0409	0.00009	0.0092	246
AJ_xslq	0.9988	1.0000	0.9528	1.0172	0.00006	0.0079	246
AA_def	1.0000	1.0000	0.9866	1.0188	0.00000	0.0018	246
AA_rates	1.0004	1.0003	0.9775	1.0225	0.00002	0.0043	246
AA_reglq	0.9992	1.0000	0.9347	1.0215	0.00004	0.0066	246
AA_xslq	0.9995	1.0000	0.9776	1.0071	0.00001	0.0033	246
A_def	1.0000	1.0000	0.9870	1.0065	0.00000	0.0014	246
A_rates	1.0003	1.0002	0.9824	1.0169	0.00001	0.0034	246
A_reglq	0.9998	1.0000	0.9441	1.0294	0.00003	0.0054	246
A_xslq	0.9995	1.0000	0.9776	1.0071	0.00001	0.0033	246
BBB_def	0.9999	1.0000	0.9952	1.0030	0.00000	0.0009	246
BBB_rates	1.0001	1.0000	0.9827	1.0273	0.00002	0.0042	246
BBB_reglq	1.0000	0.9999	0.9785	1.0248	0.00002	0.0047	246
BBB_xslq	0.9995	1.0000	0.9776	1.0071	0.00001	0.0033	246
BBBm_def	1.0000	1.0000	0.9953	1.0029	0.00000	0.0008	246
BBBm_rates	0.9999	1.0000	0.9877	1.0059	0.00000	0.0012	246
BBBm_reglq	0.9993	1.0000	0.9467	1.0170	0.00003	0.0056	246
BBBm_xslq	0.9996	1.0000	0.9665	1.0092	0.00001	0.0033	246

This table summarizes the observations of intraday data at the close of the trading day 4:15:00ET for 246 trading days during the pandemic (4/2020 - 4/2021). The first column provides the abbreviation for the variable name. The VIX is the CBOE volatility index. This is followed by the prices of 25 REITs with the name a composite made up of the property type (industrial (IN), hotel/lodging (LO), multifamily (MF), mixed use/other (OT), office (OF) and retail (RT)) and the REIT's stock market ticker. Following the REITs are US Treasury yields with ticker representing the 4 maturities of 3 month, 5 year, 10 year and 30 year. The remaining values are the simulated risk partitions indexed in Christopoulos and Jarrow (2018) in cumulative change form. The ticker is a composite of the 4 types of risk partitions default (def), interest rates (rates), liquidity (reglq) and excess liquidity (xslq) combined with the credit rating class names of AAA, AJ, AA, A, BBB and BBB- (BBBm). Each of the columns to the right of the ticker report statistics across all 246 observations.

In Tables 17 and 18 we compress these statistically for two (out of 1560) points during the trading day: the opening bell values (9:30:15) and closing bell values (4:15:00), respectively across the 246 trading days in our sample. The tables corroborate the visual insights from the cross-sectional plots: The opening is indeed much more volatile than the closing, for the risk partitions as measured by the standard deviation (stdev). Opening volatilities for the cumulative changes in proportions of all risk partitions are categorically higher than the closing volatilities for those values. Additionally, the VIX and interest rates also exhibit categorically higher opening volatility compared with closing volatility. Finally, REITs also exhibit mostly higher volatility at the open than the close, but not categorically with only 18 of the 25 REITs exhibiting higher volatility at the open in the cross-section with and 7 of the 25 exhibiting lower volatility at the open in the cross-section. The differences in magnitude between the theoretical risk partitions and the publicly traded securities and indices is not completely surprising. The risk partition measures represent cumulative changes in proportions from the start of the day. In contrast, the pricing for the publicly traded objects are just prices and not changes of prices. What is important for our purposes is the insight of higher volatility of signals at the open than the close.

4.2.3 REIT trading strategies with the Intraday Model

While we validate the Daily Model pre-Covid with actual data, during the Covid period we do not have CMBX risk premia with which to calibrate the model or to backtest as was done in the earlier work. As previously mentioned, CMBX as an OTC product does not trade electronically.[15] That means that the estimated risk partitions update more rapidly than the actual CMBX securities and their underlying CMBS collateral. Nevertheless, those estimates reflect theoretical partitions under risk neutral arguments as described in the previous studies of Christopoulos (2017) and Christopoulos and Jarrow (2018). In those studies the CMBX risk indices gave many insights into the buy/sell decision-making within the CMBX market. But perhaps with greater updated frequency, the risk partitions give insights also into related products with matching trading frequency. One such product are publicly traded REITs. Publicly traded REITs trade on electronic equity exchanges and thus have a similar frequency of price updates as our intraday risk measures. Additionally

[15] For further information see Appendix A.1.

REITs are well distributed across retail and institutional investors alike. Finally, REITs, like CMBS and CMBX, have similar underlying exposure to commercial real estate risk, and so estimated CMBX risk partitions may carry relevance to REIT pricing in the marketplace. With our trading tests we section seek to exploit phenomena observed within estimated CMBX risk partitions with respect to REIT pricing efficiency.

4.2.4 Scope and notation for trading strategies

Finally, unlike default, liquidity and excess liquidity partitions, the interest rate risk partition is a bit more difficult to interpret in the trading context with respect to REITs. Higher interest rate risk compensation for higher future rate volatility may affect REITs differently, depending on leverage and asset composition. To keep things tractable, we focus our trading strategies with REITs to the risk partitions of default, liquidity and excess liquidity. Additionally, to simplify the notation for the trading signals discussed below, for the u-th trading day the risk partition, $W_{jkut} \equiv y_{jku}(t)$ as defined in Eq. (29) with j risk partitions of k credit rating classes, u trading days and t intraday trading times. Additionally, we are fusing the risk partition W with REITs where $\iota \in [1, 25]$ is used to represent one of the 25 REITs in our sample.

Trading Signal 1 Let $\Delta R_{\iota ut}$ be defined as

$$\Delta R_{\iota ut} = \frac{R_{\iota ut}}{R_{\iota t=4:15:00pm,u-1}} \tag{30}$$

which represents the proportion of cumulative changes in REIT prices from the close of the prior trading day $t =$4:15:00 pm to the time of trade execution for the ι-th REIT.

With this, we construct the Trading Signal 1, $L^1_{\iota jkut}$, defined as

$$L^1_{\iota jkut} = |\Delta R_{\iota ut} - W_{jkut}| \tag{31}$$

which captures the absolute value of the difference between the observed proportional cumulative change in prices for the ι-th REIT, $\Delta R_{\iota ut}$, and the cumulative change in prices for one of the risk partitions, W_{jkut}.[16]

[16] Recall that $W_{jkut} \equiv y_{jk}(t)$ as defined in Eq. (29) is a cumulative proportional change in the risk

As an absolute value of differences in cumulative changes, $L^1_{\iota jkut}$ gives the magnitude of differences. If those differences are influenced by liquidity effects we appear to capture in the cross section, trading opportunities may arise. Eq. (30) does not provide a direction for the trading strategy. We borrow from differences between fair value estimates and market prices where large differences *may* indicate buying opportunities. Large differences necessarily correspond to differences in the theoretical risk measures and perspectives on risk projected by market actors onto market prices. While there is a theoretical basis for long/short trading direction in comparisons between fair value and market prices[17], the elements $\Delta R_{\iota ut}$ and W_{jkut} of Eq. (31) do not carry such ex-ante trade directional indications.

Trading Signal 2 We address some arbitrariness in Trade Signal 1's lack of direction with our second trading signal:

$$L^2_{\iota jkut} = \frac{\Delta R_{\iota ut}}{W_{jkut}} \tag{32}$$

This proportion $L^2_{\iota jkut}$ has several interesting interpretations and clear theoretical interpretations with respect to direction.

As noted in Christopoulos (2017), both the liquidity and excess liquidity measures have the interpretation of liquidity availability. And thus large values for those risk neutral measures (which are then projected onto market spreads) suggest that compensation in market spreads exceeds (wider than) what is required fair compensation from the perspective of the model technology. In the case of the current technology which does not benefit from market prices to project onto, the risk composition estimated reflect cumulative intraday changes in the measure from the opening bell. As such, higher values for cumulative changes intraday in the liquidity and excess liquidity pricing measures, suggest 'cheaper' pricing for the underlying derivatives and potentially a healthier commercial real estate environment. The opposite is true for default risk pricing. There, higher cumulative values for the pricing of default indicates greater default risk while lower cumulative values suggest lower default risk. We capture the interpretation above in the proportional trading signal, $L^2_{\iota jkut}$ and

partition, intraday.

[17]See, for example, the relative value measure theta defined in Eq. (23) in Christopoulos and Jarrow (2018) where fair value prices greater/less than market prices indicate buy/sell signals, among others.

summarize below in Table 19 fifteen numerical examples which capture all types of changes in the components of the signal. Although the trading strategy involve the ratio of 1 the statistical analysis shows the mean and median for the ratios to be strongly clustered about 1, thereby justifying this choice.

Table 19: Trade Signal Examples

Ex #	type	ΔR	W	$L = \Delta R/W$	W = XSLQ	W= LQ	W= DEF
1	$\Delta R\uparrow$, $W\uparrow$	1.250	1.250	1.0000	no trade	no trade	no trade
2	$\Delta R\downarrow$, $W\downarrow$	0.990	0.990	1.0000	no trade	no trade	no trade
3	$\Delta R\uparrow$, $W\uparrow$	1.250	1.200	1.0417	sell	sell	buy
4	$\Delta R\downarrow$, $W\downarrow$	0.990	0.980	1.0102	sell	sell	buy
5	$\Delta R\uparrow$, $W0$	1.500	1.000	1.5000	sell	sell	buy
6	$\Delta R\uparrow$, $W\downarrow$	1.200	0.999	1.2012	sell	sell	buy
7	$\Delta R0$, $W\downarrow$	1.000	0.900	1.1111	sell	sell	buy
8	$\Delta R0$, $W0$	1.000	1.000	1.0000	no trade	no trade	no trade
9	$\Delta R0$, $W\uparrow$	1.000	1.500	0.6667	buy	buy	sell
10	$\Delta R\downarrow$, $W\uparrow$	0.800	1.500	0.5333	buy	buy	sell
11	$\Delta R\downarrow$, $W0$	0.900	1.000	0.9000	buy	buy	sell
12	$\Delta R\downarrow$, $W\downarrow$	0.960	0.970	0.9897	buy	buy	sell
13	$\Delta R\uparrow$, $W\uparrow$	1.020	1.100	0.9273	buy	buy	sell
14	$\Delta R\downarrow$, $W\downarrow$	0.960	0.960	1.0000	no trade	no trade	no trade
15	$\Delta R\uparrow$, $W\uparrow$	1.020	1.020	1.0000	no trade	no trade	no trade

This table summarizes the logic underlying the direction (long/buy or short/sell) for Trading Signal 2, $L^2_{\iota jkut}$. The 15 examples show possible trading signal impact, numerically, for different cumulative changes in REIT prices, ΔR, and different cumulative changes in the risk partition, W. The rows capture the different examples and the columns the combination of changes (type), the individual changes ($\Delta R$and W) the signal ($L = \Delta R/W$), and the different types of risk partition tested in the trading strategies: excess liquidity (W = XSLQ), liquidity (W = LQ), and default (W = DEF). The rowwise indication of 'no trade' indicates that no valid trade signal is observed for L at that time. The indication of 'sell' is a signal to allocate the REIT corresponding to R into the short portfolio, while an indication of 'buy' is a signal to allocate to the REIT correspond to R into the long portfolio.

For liquidity and excess liquidity measures, higher values of $L^2_{\iota jkut} > 1$ correspond to 'sell' signals while lower values of $L^2_{\iota jkut} < 1$ correspond to 'buy' signals. Values of $L^2_{\iota jkut} = 1$ indicate no trade signal. It is important to see that the relative rate of change between the numerator and denominator come into play and the signal picks up these subtleties. For example, consider in Table 19 Examples 3 and 13, $\Delta R \uparrow, W\uparrow$. Both instances correspond to simultaneous increases in both the numerator and the denominator, indicated by the upward pointing arrows. In Example 3, the cumulative proportional liquidity measure W is increasing more slowly (1.20) than the cumulative proportional REIT price change ΔR (1.25). Since liquidity availability is lagging price increases, the trade signal suggests 'sell' as prices are moving higher than justified relative to the liquidity measure ($L = 1.25/1.20 = 1.0417 > 1 \rightarrow$ 'sell'). In contrast, in Example 13, also $\Delta R \uparrow, W\uparrow$, since liquidity (1.10) is increasing more rapidly than the cumulative price increases in the REIT (1.02), the trade signal suggests a buying opportunity ($L = 1.02/1.10 = 0.9273 < 1 \rightarrow$ 'buy'). Finally, while liquidity and excess liquidity availability generate the same directional signalling, the default risk measure generates exactly the opposite signal, because higher default risk decompositions correspond to higher default risk as noted in Christopoulos (2017).[18]

Trading Strategies with Trading Signals 1 and 2 On each day we consider the set of all 25 REITs with the Trading Signals $L^1_{\iota jkut}$ and $L^2_{\iota jkut}$ for all risk partitions for all credits. We 'buy' securities into the long portfolio and 'sell' securities into the short portfolio. We assume execution at mid-market prices for the REITs when we enter and exit trades and do not, in this initial study, make adjustments for bid/ask spreads of REITs which were not provided. We execute all our trading (long and short positions) at the first trade of the day at approximately t =9:30:15 am EST. We unwind all positions at end of the day at the close of the day approximately t =4:15:00pm EST.[19] For all strategies we rank order the REITs based on the values of $L^1_{\iota jkut}$ and $L^2_{\iota jkut}$ and buy or sell them instantaneously into their respective portfolios at the price which contributes to the signals. In this paper, long and short portfolios are assumed to make price weighted contributions to the long/short portfolio components with the daily returns on long/short components equally weighted.

Trading Strategy 1 For Trading Strategy 1 we use trade signal 1, $L^1_{\iota jkut}$. We consider two alternatives: i.) an even split of 12 long and 12 short positions assumed to make and ii.) a top-3/bottom-3 strategy (close to deciles) with an even split of 3 long and 3 short positions. The selection based on the rank order of the REITs based on the signals assumes that larger differences (greater magnitude) between $\Delta R_{\iota ut}$ and W_{jkut} represent buying opportunities (long portfolio) and smaller differences selling opportunities (short portfolio).

Trading Strategy 2 For Trading Strategy 2 we use $L^2_{\iota jkut}$. We consider the rank order of the REITs based on the signals with interpretations given in Table 19. For liquidity and excess liquidity risk availability portfolios, for those REITs with values of $L^2_{\iota jkut} > 1$ we 'sell' those securities in the short portfolio; and for those REITs with values $L^2_{\iota jkut} < 1$ we 'buy' those securities into the long portfolio. In the case of the default risk partition portfolio we

[18]While we also capture the rates uncertainty partition we do not focus on it in these strategies which is left to future research.

[19]On occasion there are small differences of a few seconds due to latency. All trading times are time stamped.

buy (long) REITs with values of $L^2_{\imath jkut} < 1$ and sell (short) REITs with values of $L^2_{\imath jkut} > 1$. Values of $L^2_{\imath jkut} = 1$ indicate no trade signal in all cases.

It is certainly the case in this strategy that on any given day we may be 100% directionally long or short, but we are never more than 50% weighted long/short. In the instances where 100% of the values $L^2_{\imath jkut}$ are in the same direction (Eg. all > 1 or all< 1), then the corresponding portfolio is allocated 50% to the direction given by the uniform signal on that day and 50% into cash (0% Return). For example, if all 25 REITs had a buy-signal based on $L^2_{\imath jkut}$ then all 25 would be purchased into the long position. The return for the day would be calculated based on the change in value between 9:30:15am (when they were purchased) and 4:15:00pm (when they were sold). However, in such cases 50% of the portfolio is said to have been moved to cash with assumed 0% return for that day. On all other days, when the signals are mixed 50% of the returns are generated by the long leg of the portfolio and 50% are generated by the short leg of the portfolio, regardless of how many signals indicate long and short on such date. The risk management of the portfolio is enforced in cases where only one direction is signalled, with a 50% allocation to cash imposed. Additionally, the time horizon of mandatory unwind of all positions by the end of the day ensures that each portfolio horizon is only one trading session. The obvious artifice is the forcing trades to close at the end of the day instead exploiting the intraday changes observable in Figures 5, 6, 7 and 8. Investigation of such tactical intraday exploitation of CMBX risk partition pricing is outside the scope of this paper and left to future work.

4.2.5 Results from Trading Strategies 1 and 2

The trading strategies were implemented for excess liquidity, liquidity and default risk measures for all ratings classes summarized in Table 20. The best results were found with the excess liquidity risk strategy returns of 48.74% for AAA using L1 1010 (Panel A). The worst result was -50.13% 12-month returns for A using L2 (Panel D). Sharpe ratios, H_ω, for the ω-th different trading strategies[20] with $\omega \in [1, 54]$ trading strategies were calculated as

$$H_\omega = \frac{R_\omega - R_\lambda}{\sigma_\omega \times \sqrt{240}} \tag{33}$$

[20]See Sharpe (1994).

The square root of the 240 trading days is the required adjustment for daily standard deviations for 240 trading days in our sample period. The cumulative return for the specific day trading strategy is denoted, R_ω, while R_λ is the cumulative return for the long-only portfolio. The long-only portfolio, like the day trading strategies, is also a day traded portfolio executed with buys and sells contemporaneous with the long/short portfolio. The standard deviation of the daily returns for the day trading strategy portfolio is denoted as σ_ω.

Table 20: Cumulative returns for Trading Strategies 1 and 2

Panel A: AAA	L1 5050	L1 1010	L2
XSLiquidity	19.21%	48.74%	9.09%
stdev port	0.61%	1.18%	0.78%
Sharpe Ratio	3.61	3.50	1.99
Liquidity	2.02%	-5.63%	37.69%
stdev port	0.62%	1.11%	1.09%
Sharpe Ratio	1.77	0.54	3.12
Default	-3.52%	-18.21%	11.08%
stdev port	0.63%	1.20%	0.80%
Sharpe Ratio	1.17	-0.17	2.11

Panel B: AJ	L1 5050	L1 1010	L2
XSLiquidity	-8.22%	-12.47%	41.37%
stdev port	0.63%	1.26%	0.71%
Sharpe Ratio	0.69	0.13	5.09
Liquidity	6.80%	16.93%	22.80%
stdev port	0.63%	1.23%	1.09%
Sharpe Ratio	2.24	1.67	2.23
Default	1.53%	-8.01%	17.08%
stdev port	0.64%	1.28%	0.72%
Sharpe Ratio	1.66	0.35	2.87

Panel C: AA	L1 5050	L1 1010	L2
XSLiquidity	0.73%	-5.79%	37.62%
stdev port	0.62%	1.22%	0.78%
Sharpe Ratio	1.64	0.48	4.37
Liquidity	5.55%	3.58%	14.34%
stdev port	0.64%	1.22%	0.74%
Sharpe Ratio	2.09	0.98	2.57
Default	1.09%	-0.09%	-5.23%
stdev port	0.63%	1.23%	0.79%
Sharpe Ratio	1.64	0.78	0.80

Panel D: A	L1 5050	L1 1010	L2
XSLiquidity	0.73%	-5.79%	37.62%
stdev port	0.62%	1.22%	0.78%
Sharpe Ratio	1.64	0.48	4.37
Liquidity	7.14%	19.81%	34.89%
stdev port	0.64%	1.20%	0.75%
Sharpe Ratio	2.24	1.88	4.30
Default	12.38%	6.23%	-13.02%
stdev port	0.62%	1.24%	0.73%
Sharpe Ratio	2.86	1.10	0.17

Panel E: BBB	L1 5050	L1 1010	L2
XSLiquidity	0.73%	-5.79%	37.62%
stdev port	0.62%	1.22%	0.78%
Sharpe Ratio	1.64	0.48	4.37
Liquidity	4.32%	-2.47%	39.95%
stdev port	0.64%	1.23%	0.77%
Sharpe Ratio	1.95	0.66	4.63
Default	16.12%	13.10%	-34.67%
stdev port	0.61%	1.24%	0.84%
Sharpe Ratio	3.27	1.47	-1.52

Panel F: BBB-	L1 5050	L1 1010	L2
XSLiquidity	11.00%	18.50%	15.09%
stdev port	0.61%	0.97%	0.76%
Sharpe Ratio	2.75	2.23	2.57
Liquidity	-0.01%	-1.59%	21.03%
stdev port	0.63%	1.25%	0.72%
Sharpe Ratio	1.53	0.69	3.24
Default	6.23%	11.98%	-32.53%
stdev port	0.64%	1.20%	0.86%
Sharpe Ratio	2.14	1.44	-1.31

This table provides the cumulative returns over 240 consecutive trading days over the sample period for Trading Strategies 1 and 2 using the four different risk partitions of default, liquidity and excess liquidity as embedded within Eqs. (31 and 32). The columns show results for Trading Strategy 1 for upper and lower 50% (L1 5050), Trading Strategy 1 for upper and lower deciles (L1 1010), and Trading Strategy 2. Each of the rows within the Panels A thru E show the returns for the risk partitions in the order Excess (XS) Liquidity, Liquidity, and Default with the standard deviation for the portfolio and the Sharpe Ratio for the portfolio.***/**/*/’ correspond to 0.1%, 1%, 5% and 10% levels of significance. Panel A, shows the results for AAA, Panel B shows the results for AJ, Panel C shows the results for AA, Panel D shows the results for A, Panel E shows the results for BBB and Panel F shows the results for BBB-.

The long-only portfolio has 100% allocation to the entire set of REITs in our sample, bought at 9:30am and sold at 4:00pm on each trading day. The return on the long-only portfolio over the sample period was -14.97%. For portfolios with positive returns, the portfolio Sharpe ratios were quite strong with values ranging from 0.98 to 5.09. In general, the Sharpe ratios for the L2 strategies were higher than either of the L1 strategies. Concentrating on L2 trading signals for the 24 REIT relative value strategies (4 per credit rating class), for excess liquidity and liquidity trading we see categorically positive positive returns ranging from 9.09% to 41.37% for all credit rating classes. Interestingly for the default pricing driven signals, positive returns of 11.08% and 17.08% are captured only for the AAA and AJ classes, respectively. Additionally, the only credit rating class to exhibit positive market risk signal (rates) with respect to REIT relative value is AAA (15.69%). These findings suggest that the L2 liquidity and excess liquidity trading signals with respect to REIT relative value provide more consistent and positive insights for this exercise.[21]

4.2.6 ICAPM

To test for skill in the trading strategies we use the ICAPM of Merton (1990). We choose this approach as a standard approach for equity asset pricing tests of skills. An alternative approach introduced by Acharya and Pedersen (2005) which includes the Amihud (2002) ILLIQ measure. However, ILLIQ is dependent on observed volumes. Since no observed volumes were provided for CMBX, ILLIQ cannot be implemented as an ICAPM factor in this study. However, the key findings of Acharya and Pedersen (2005) concerning flight to liquidity do appear to be confirmed by analyses of our trading strategies that exploit the estimated reduced form measures of liqudity and excess liquidity.

Following Christopoulos and Jarrow (2018) the final regression model to test for abnormal returns in our trading strategies is given by:

$$R_{\omega u} - R_{\lambda u} = \alpha + \sum_{i=2}^{M} \beta_{\omega i}(R_{iu} - r_u) + \varepsilon_u \tag{34}$$

with $u \in [1, 240]$ trading days, $R_{\omega u}$ the daily returns of the trading strategy portfolio, $R_{\lambda u}$ the returns of the long-only trading portfolio, R_{iu} the i-th portfolio equity risk factor, and r_u the risk-free rate. Positive and significant α implies these trading strategies generate abnormal returns. We use the standard risk factors to evaluate equity based trading strategies introduced in Fama and French (1993) including: (i) the market portfolio, (ii) the SMB

[21] Tables of all trades for all trading strategies are available upon request.

equity index, and (iii) the HML index as well as the MOM risk factor introduced in Carhart (1997). We test 54 different trading strategies summarized in Table 21. For each of the six credit rating classes we conduct nine daily trading strategies across 240 trading dates from $u = 4/8/2020$ thru $u = 3/31/2021$. Trading Strategy 1 implements 50th percentile ('5050') and decile ('1010') variations.

Table 21: ICAPM Results

Panel A, AAA	α	Mkt-Rf	MOM	SMB	HML	F	Adj-Rsq	N
L1_XS_5050	0.0021865**	-0.007037***	-0.001197	0.0003331	-0.0053299***	50.4	0.4376	240
	0.0008302	0.0007039	0.0008252	0.0010443	0.0011298	0.00		
L1_XS_1010	0.0040237*	-0.014177***	-0.0022281	0.0006341	-0.0100508***	47.44	0.3746	240
	(0.0016415)	(0.0013917)	(0.0016317)	(0.0020649)	(0.0022338)	0.00		
L1_Lq_5050	0.0037889*	-0.0114627***	-0.0008965	-0.0012026	-0.0078067***	38.66	0.3187	240
	(0.0015924)	(0.0013502)	(0.0015829)	(0.0020032)	(0.0021671)	0.00		
L1_Lq_1010	0.0029496.	-0.0107726***	-0.002226	-0.0008903	-0.0092229***	29.54	0.1885	240
	(0.0017257)	(0.0014632)	(0.0017155)	(0.0021709)	(0.0023485)	0.00		
L1_Def_5050	0.004655**	-0.013347***	-0.001268	-0.000568	-0.008402***	48.62	0.4092	240
	(0.001554)	(0.001318)	(0.001545)	(0.001955)	(0.002115)	0.00		
L1_Def_1010	0.0055647***	-0.0113874***	-0.0005711	-0.0013482	-0.0082612***	40.18	0.3037	240
	(0.0016388)	(0.0013895)	(0.0016291)	(0.0020616)	(0.0022302)	0.00		
L2_XS	0.004437**	-0.011797***	-0.001651	-0.00105	-0.008825***	39.87	0.3187	240
	(0.001599)	(0.001355)	(0.001589)	(0.002011)	(0.002176)	0.00		
L2_Lq	0.0038399*	-0.0137208***	-0.0012093	0.0007149	-0.0092415***	54.21	0.476	240
	(0.0015339)	(0.0013006)	(0.0015248)	(0.0019296)	(0.0020874)	0.00		
L2_Def	0.003991*	-0.015079***	-0.001129	0.00106	-0.009997***	52.45	0.4191	240
	(0.001714)	(0.001454)	(0.001704)	(0.002157)	(0.002333)	0.00		
Panel B, AJ	α	Mkt-Rf	MOM	SMB	HML	F	Adj-Rsq	N
L1_XS_5050	0.0043294**	-0.0130814***	-0.0016926	0.0004567	-0.009509***	54.74	0.4741	240
	(0.0014628)	(0.0012403)	(0.0014541)	(0.0018402)	(0.0019907)	0.00		
L1_XS_1010	0.005038**	-0.014427***	-0.001551	0.001179	-0.010328***	55.11	0.4243	240
	(0.00161)	(0.001365)	(0.0016)	(0.002025)	(0.00219)	0.00		
L1_Lq_5050	0.0038897*	-0.0115571***	-0.0019515	-0.0005213	-0.0094571***	42.6	0.3557	240
	(0.0015353)	(0.0013017)	(0.0015261)	(0.0019313)	(0.0020892)	0.00		
L1_Lq_1010	0.0033418*	-0.0117045***	-0.0031924.	0.0003376	-0.0109727***	34.93	0.2388	240
	(0.0016865)	(0.0014299)	(0.0016765)	(0.0021216)	(0.0022951)	0.00		
L1_Def_5050	0.0057627***	-0.0123883***	-0.0015501	-0.0006195	-0.00886***	43.27	0.3616	240
	(0.0015814)	(0.0013408)	(0.0015719)	(0.0019893)	(0.002152)	0.00		
L1_Def_1010	0.004989**	-0.012098***	-0.001468	-0.001534	-0.009482***	41.29	0.3222	240
	(0.001688)	(0.001431)	(0.001678)	(0.002124)	(0.002297)	0.00		
L2_XS	0.004683**	-0.011807***	-0.002287	-0.001453	-0.009541***	41.04	0.3391	240
	(0.001586)	(0.001345)	(0.001576)	(0.001995)	(0.002158)	0.00		
L2_Lq	0.004194**	-0.01331***	-0.0009555	0.00002154	-0.008433***	53.18	0.4646	240
	(0.0015)	(0.001272)	(0.001491)	(0.001887)	(0.002041)	0.00		
L2_Def	0.004079*	-0.01402***	-0.001466	-0.00004531	-0.008893***	45.21	0.3462	240
	(0.00167)	(0.001416)	(0.00166)	(0.002101)	(0.002273)	0.00		
Panel C, AA	α	Mkt-Rf	MOM	SMB	HML	F	Adj-Rsq	N
L1_XS_5050	0.004027*	-0.0116518***	-0.0023509	-0.0008969	-0.0094466***	38.3	0.3151	240
	(0.001595)	(0.0013524)	(0.0015855)	(0.0020065)	(0.0021706)	0.00		
L1_XS_1010	0.004226*	-0.012386***	-0.002168	-0.001093	-0.010095***	34.81	0.2644	240
	(0.001826)	(0.001548)	(0.001815)	(0.002296)	(0.002484)	0.00		
L1_Lq_5050	0.0041002**	-0.0123558***	-0.0013576	-0.0005149	-0.0087784***	50.84	0.4356	240
	(0.0014657)	(0.0012427)	(0.0014569)	(0.0018438)	(0.0019946)	0.00		
L1_Lq_1010	0.0040813**	-0.0131186***	-0.0019544	-0.0001837	-0.0093178***	47.89	0.3429	240
	(0.0015437)	(0.0013088)	(0.0015345)	(0.0019419)	(0.0021007)	0.00		
L1_Def_5050	0.005408***	-0.0117627***	-0.0017786	-0.0001356	-0.0089674***	38.24	0.3021	240
	(0.0015995)	(0.0013561)	(0.0015899)	(0.0020121)	(0.0021766)	0.00		
L1_Def_1010	0.004603**	-0.011206***	-0.001249	-0.002266	-0.008399***	41.16	0.3384	240
	(0.001572)	(0.001333)	(0.001563)	(0.001978)	(0.002139)	0.00		
L2_XS	0.0038622*	-0.0125738***	-0.0021869	-0.0009717	-0.0098577***	41.67	0.3486	240
	(0.0016568)	(0.0014047)	(0.0016469)	(0.0020842)	(0.0022547)	0.00		
L2_Lq	0.004194**	-0.01331***	-0.0009555	0.00002154	-0.008433***	53.18	0.4646	240
	(0.0015)	(0.001272)	(0.001491)	(0.001887)	(0.002041)	0.00		
L2_Def	0.004079*	-0.01402***	-0.001466	-0.00004531	-0.008893***	45.21	0.3462	240
	(0.00167)	(0.001416)	(0.00166)	(0.002101)	(0.002273)	0.00		

Panel D, A	α	Mkt-Rf	MOM	SMB	HML	F	Adj-Rsq	N
L1_XS_5050	0.0042909**	-0.0125106***	-0.0016312	-0.0002349	-0.0094443***	46.2	0.3982	240
	(0.0015688)	(0.0013301)	(0.0015594)	(0.0019735)	(0.0021348)	0.00		
L1_XS_1010	0.004902**	-0.012716***	-0.0009979	-0.0001228	-0.0091501***	39.08	0.3026	240
	(0.0017638)	(0.0014954)	(0.0017532)	(0.0022188)	(0.0024002)	0.00		
L1_Lq_5050	0.00454**	-0.012023***	-0.001855	-0.001405	-0.00898***	47.11	0.4028	240
	(0.001491)	(0.001264)	(0.001482)	(0.001876)	(0.002029)	0.00		
L1_Lq_1010	0.004231**	-0.012184***	-0.002705.	-0.00114	-0.009916***	41.87	0.2924	240
	(0.00158)	(0.00134)	(0.001571)	(0.001988)	(0.002151)	0.00		
L1_Def_5050	0.005408***	-0.0117627***	-0.0017786	-0.0001356	-0.0089674***	38.24	0.3021	240
	(0.0015995)	(0.0013561)	(0.0015899)	(0.0020121)	(0.0021766)	0.00		
L1_Def_1010	0.005718***	-0.013031***	-0.002128	-0.001359	-0.010209***	47.62	0.4112	240
	(0.00163)	(0.001382)	(0.00162)	(0.00205)	(0.002218)	0.00		
L2_XS	0.003497*	-0.012009***	-0.002309	-0.001462	-0.010016***	42.67	0.3593	240
	(0.00161)	(0.001365)	(0.0016)	(0.002025)	(0.002191)	0.00		
L2_Lq	0.004194**	-0.01331***	-0.0009555	0.00002154	-0.008433***	53.18	0.4646	240
	(0.0015)	(0.001272)	(0.001491)	(0.001887)	(0.002041)	0.00		
L2_Def	0.004079*	-0.01402***	-0.001466	-0.00004531	-0.008893***	45.21	0.3462	240
	(0.00167)	(0.001416)	(0.00166)	(0.002101)	(0.002273)	0.00		
Panel E, BBB	α	Mkt-Rf	MOM	SMB	HML	F	Adj-Rsq	N
L1_XS_5050	0.0038162*	-0.0117119***	-0.0024842	0.0005267	-0.0100017***	35.93	0.2936	240
	(0.0016434)	(0.0013934)	(0.0016336)	(0.0020674)	(0.0022365)	0.00		
L1_XS_1010	0.003713*	-0.0120828***	-0.002494	0.0006448	-0.0109529***	31.68	0.2385	240
	(0.0018763)	(0.0015908)	(0.0018651)	(0.0023603)	(0.0025533)	0.00		
L1_Lq_5050	0.004766**	-0.012471***	-0.00194	-0.001338	-0.00944***	50.23	0.4355	240
	(0.001503)	(0.001274)	(0.001494)	(0.00189)	(0.002045)	0.00		
L1_Lq_1010	0.0046653**	-0.0129274***	-0.0025898	-0.0009711	-0.0099328***	42.18	0.3085	240
	(0.0016414)	(0.0013917)	(0.0016316)	(0.0020649)	(0.0022337)	0.00		
L1_Def_5050	0.005408***	-0.0117627***	-0.0017786	-0.0001356	-0.0089674***	38.24	0.3021	240
	(0.0015995)	(0.0013561)	(0.0015899)	(0.0020121)	(0.0021766)	0.00		
L1_Def_1010	0.00561***	-0.011541***	-0.001856	-0.001621	-0.009152***	44.03	0.3589	240
	(0.001526)	(0.001294)	(0.001517)	(0.00192)	(0.002077)	0.00		
L2_XS	0.0025332	-0.0120336***	-0.0023626	-0.0006994	-0.0098406***	38.99	0.3121	240
	(0.0016395)	(0.0013901)	(0.0016297)	(0.0020625)	(0.0022311)	0.00		
L2_Lq	0.004575**	-0.0128342***	-0.0006617	-0.0003317	-0.0079207***	52.7	0.459	240
	(0.0014701)	(0.0012464)	(0.0014613)	(0.0018493)	(0.0020005)	0.00		
L2_Def	0.005056**	-0.0134311***	-0.0003629	-0.0002725	-0.0077591***	51.39	0.4141	240
	(0.0015477)	(0.0013123)	(0.0015385)	(0.001947)	(0.0021063)	0.00		
Panel F, BBB-	α	Mkt-Rf	MOM	SMB	HML	F	Adj-Rsq	N
L1_XS_5050	0.004217**	-0.013495***	-0.001041	0.000192	-0.008697***	58.21	0.5052	240
	(0.001453)	(0.001232)	(0.001444)	(0.001828)	(0.001977)	0.00		
L1_XS_1010	0.004322**	-0.015282***	-0.001788	0.001642	-0.009734***	56.21	0.4296	240
	(0.00158)	(0.00134)	(0.001571)	(0.001988)	(0.00215)	0.00		
L1_Lq_5050	0.004165**	-0.011688***	-0.001834	-0.001053	-0.009192***	40.45	0.3382	240
	(0.001591)	(0.001349)	(0.001582)	(0.002002)	(0.002166)	0.00		
L1_Lq_1010	0.0043642*	-0.0116363***	-0.0019992	-0.0008662	-0.0091518***	33.42	0.2301	240
	(0.0017165)	(0.0014554)	(0.0017063)	(0.0021593)	(0.0023359)	0.00		
L1_Def_5050	0.0044753**	-0.0118264***	-0.0023004	-0.0003724	-0.0100502***	42.17	0.3458	240
	(0.0015752)	(0.0013356)	(0.0015658)	(0.0019816)	(0.0021436)	0.00		
L1_Def_1010	0.0048238**	-0.0124841***	-0.00216	-0.0006673	-0.0094188***	42.97	0.3586	240
	(0.001581)	(0.0013405)	(0.0015716)	(0.0019889)	(0.0021515)	0.00		
L2_XS	0.002663	-0.012487***	-0.003047.	-0.001407	-0.010772***	40.39	0.3351	240
	(0.001682)	(0.001426)	(0.001672)	(0.002116)	(0.002289)	0.00		
L2_Lq	0.0066207**	-0.0180265***	-0.0027309	-0.0004056	-0.0140932***	46.3	0.4231	240
	(0.0022618)	(0.0019177)	(0.0022483)	(0.0028453)	(0.003078)	0.00		
L2_Def	0.0078028***	-0.0181336***	-0.0023276	-0.0000474	-0.0137304***	43.97	0.3912	240
	(0.0023297)	(0.0019753)	(0.0023158)	(0.0029307)	(0.0031704)	0.00		

This table provides the results the ICAPM regressions of Merton (1990) with the four factors introduced by Fama and French (1993) (the market portfolio (Mkt-Rf), high minus low (HML), and small minus big (SMB) and the fourth factor of momentum (MOM) introduced by Carhart (1997). The form of the regression is $R_{\omega u} - R_{\lambda u} = \alpha + \sum_{i=2}^{M} \beta_{\omega i}(R_{iu} - r_u) + \varepsilon_u$ with the difference between the trading strategy portfolio minus the long only portfolio as the dependent variable, and the four factors as independent variables. Nine strategies are tested for each credit rating class captured in the panels. Each panel presents excess liquidity (XS) for the L1 5050 and L1 1010 strategies, Liquidity for the L1 5050 and L1 1010 strategies, and Default for the L1 5050 and L1 1010 strategies. These are followed by the three L2 strategies for Excess Liquidity (L2_XS), Liquidity (L2_LQ) and Default (L2_Def). The columns correspond to intercept alpha and each of the explanatory factors. The final three columns show the F-test value, the Adjusted R-squared value and the number of observations. The estimates, the F-test value, the Adjusted R-squared value and number of observations are in the row labelled with the trading strategy. The standard error of the estimates (in parentheses) are shown in the row immediately below the estimates and the p-value for the F-test immediately below the F-test statistics. Panel A, shows the results for AAA, Panel B shows the results for AJ, Panel C shows the results for AA, Panel D shows the results for A, Panel E shows the results for BBB and Panel F shows the results for BBB-.

Table 21 provides the ICAPM results for the 54 trading strategies. It is divided into 6 panels corresponding to credit ratings classes AAA thru BBB-. 'L1' indicates Trading

Strategy 1 while 'L2' indicates Trading Strategy 2. Standard errors are provided in parentheses below corresponding estimates. P-values are provided below F-stats.

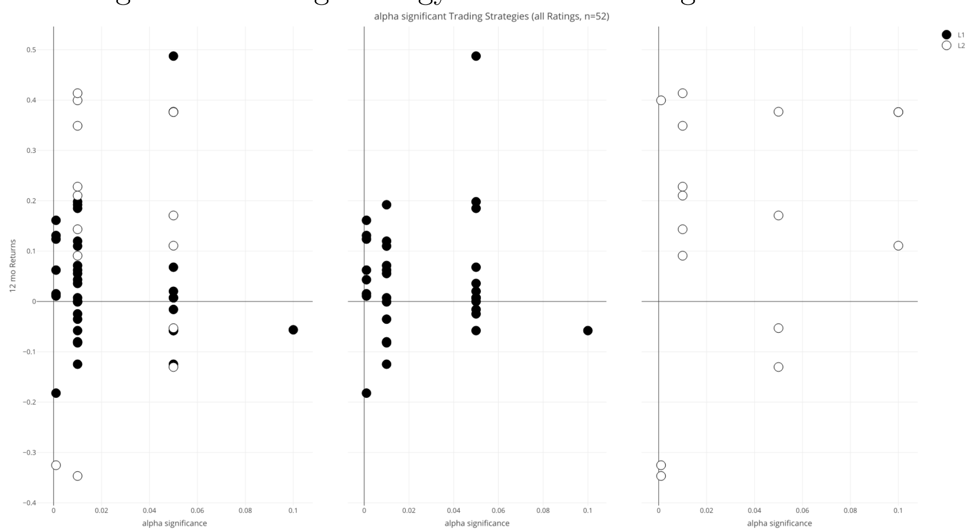
The results are quite strong. All strategies were statistically significant overall based on the F-test. Adjusted R-squared values ranged from 0.1885 to 0.5052. All strategies produced positive $\alpha's$. 52 of the 54 total trading strategies (96.30%) produced statistically significant $\alpha's$ ranging from 0.001 to 0.100 levels significance, with 35 of 54 (64.81%) producing statistically significant $\alpha's$ ranging from 0.001 to 0.010. In all strategies the market portfolio was statistically significant at the 0.001 level and negative. The MOM risk factor was generally insignificant and varied in sign. SMB varied in sign and shown to be insignificant across all strategies. HML was categorically negative exhibiting statistical significance ranging from 0.001 to 0.010. In the aggregate 65.38% of the statistically significant α strategies (34/52) also exhibited positive abnormal returns.

4.2.7 Analysis of results

Recall of 54 total strategies, 36 were L1 and 18 were L2. Of the 36 L1 strategies, all exhibited statistically significant $\alpha's$ (100.00%). Of the 18 L2 strategies, 16 exhibited statistically significant $\alpha's$ (88.89%). Interestingly for the L1 strategies, it was not categorically the case that the 1010 sub-strategies (deciles) demonstrated higher statistical significance with respect to $\alpha's$ than their corresponding 5050 strategies.

To get a better sense of the relative performance of L1 compared with L2 strategies, in Figure 9 we show the portfolio returns in Table 20 with their corresponding $\alpha's$ significance from in Table 21. Figure 9 depicts the 52 strategies that had statistically significant and positive $\alpha's$. The x-axis indicates the standard statistical significance groupings of 0.001, 0.010, 0.050 and 0.100. The y-axis indicates the 12-month cumulative return earned by the strategy. From left to right, Panel 1 shows the statistically significant α returns for the L1 and L2 strategies. Panel 2 shows the statistically significant α returns for the L1 strategies, while Panel 3 shows the statistical significant α returns for the L2 strategies. Of the significant L1 strategies, 22 of 36 showed positive returns (66.7%). In contrast, of the 16 significant L2 strategies depicted 12 showed positive returns (75.0%).

Figure 9: Trading strategy returns and α significance levels



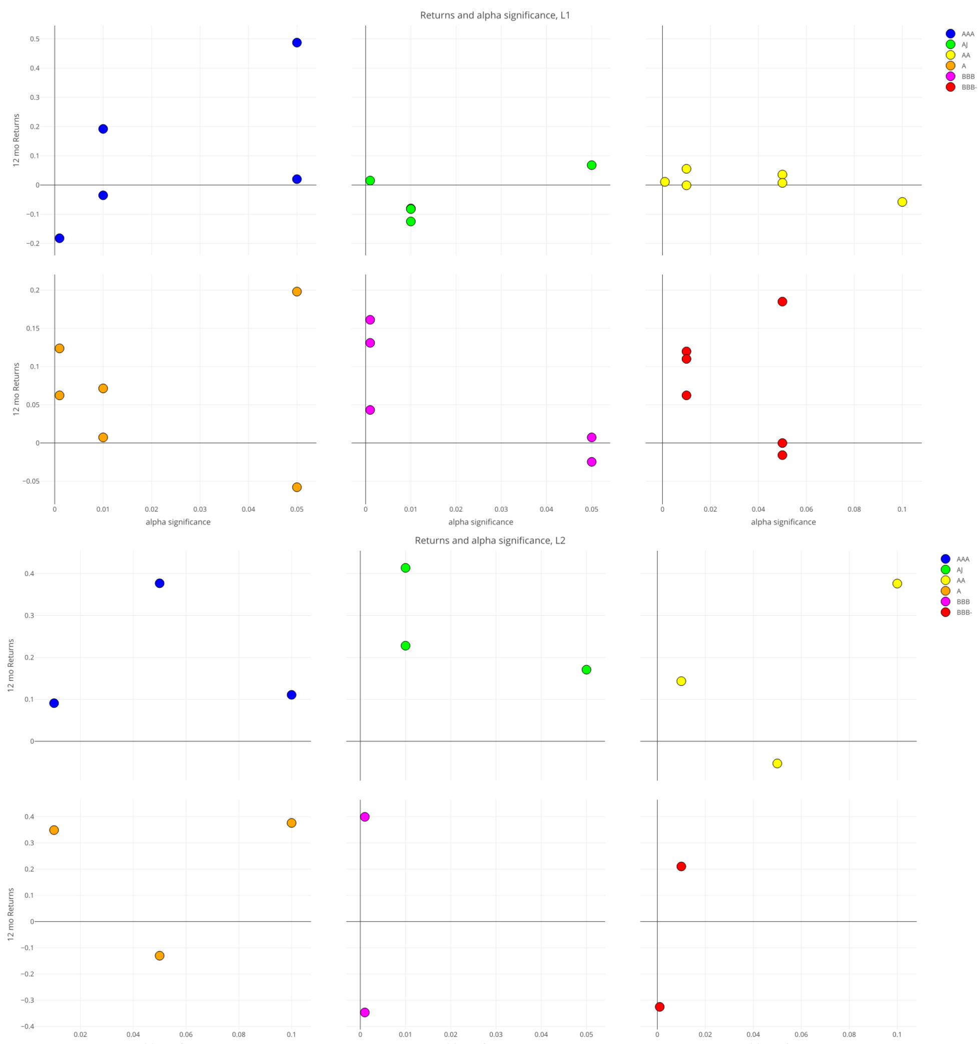
This figure depicts the 52 (out of 54 total) trading strategy returns that statistically significant and positive α's showing their α significance and the returns for those those strategies. The x-axes show α significance levels and the y-axes show the cumulative returns over the sample period. The black dots correspond to L1 strategies, while the white dots correspond to L2 strategies.

This comparatively greater proportion of positive returns coupled with positive and significant α is echoed when we break down returns by credit rating class as seen in Figure 10. The plots are read from left to right by descending credit rating class beginning with AAA (blue) in the top left and BBB- (red) in the bottom right. The top chart of 6 panels depict the statistically significant α L1 trading strategies. The bottom chart of 6 panels depicts the statistically significant L2 trading strategies. L2 strategies for the higher credits generally outperform with respect to returns at higher α significance levels for AAA, AJ, AA, and A classes. For BBB and BBB- classes, L1 appears to relatively outperform L2 in the REIT strategies tested. Across all strategies and credits, the L2 strategies for the AJ class (green) appear to perform best while L1 strategies, also for AJ class, appear to perform worst. Additionally, L1 shows a proportionately larger number of statistically significant strategies driven by BBB and BBB- classes than L2 which relatively dominates on a proportional basis within the higher credit classes. This capital structure differences are consistent with the earlier literature[22] that finds idiosyncratic risk of collateral in CMBX is comparatively more important for lower credit rated tranches than higher rated tranches. Interestingly, as noted in An, Deng, Nichols and Sanders (2015) the pricing of credit risk alone is insufficient to

[22]See for example Riddiough and Zhu (2016), among others.

explain subordination impact on pricing. As such, the logic underlying L2 trading strategies, summarized in Table 19, may be somewhat obscured for BBB and BBB- classes. While the W_{jkut} values in Eq. (32) signals are picking up risk tranching effects, and are differentiating across different types of risks, they nevertheless are not directly pricing the idiosyncratic risks of CMBX collateral with reduced form simulation and current data as shown in Christopoulos (2017) and Christopoulos and Jarrow (2018). This is a limitation to be explored in future work where computational costs and data are made available.

Figure 10: Trading strategy returns and alpha significance levels by ratings



This figure depicts the 52 (out of 54 total) trading strategy returns with statistically significant and positive α's showing their α significance and the returns for those those strategies, broken out by credit rating class. The x-axes show α significance levels and the y-axes show the cumulative returns over the sample period. The top panel of 6 sub charts capture the 36 results for the L1 trading strategies, while the bottom panel of 6 subcharts capture the 16 results for the L2 trading strategies. The credit ratings begin with AAA in the upper left corner and descend rowwise from left to right ending with BBB- in the bottom right corner.

Table 22: Summary of L1 and L2 ICAPM significant strategies (all ratings)

Panel A: L2	>=0	<0	significant	insignificant	all	% >= & significant	avg return	median ret	min ret	max ret
XS	4	0	4	2	6	100%	31.43%	37.62%	9.09%	41.37%
Lq	6	0	6	0	6	100%	28.45%	28.85%	14.34%	39.95%
Def	2	4	6	0	6	33%	-9.56%	-9.16%	-34.67%	17.08%
Panel B: L1 1010	**>=0**	**<0**	**significant**	**insignificant**	**all**	**% >= & significant**	**avg return**	**median ret**	**min ret**	**max ret**
XS	2	4	6	0	6	33%	6.23%	-5.79%	-12.47%	48.74%
Lq	2	4	6	0	6	33%	0.21%	-2.03%	-12.47%	19.81%
Def	3	3	6	0	6	50%	0.83%	3.07%	-18.21%	13.10%
Panel C: L1 5050	**>=0**	**<0**	**significant**	**insignificant**	**all**	**% >= & significant**	**avg return**	**median ret**	**min ret**	**max ret**
XS	5	1	6	0	6	83%	4.03%	0.73%	-8.22%	19.21%
Lq	5	1	6	0	6	83%	4.30%	4.94%	-0.01%	7.14%
Def	5	1	6	0	6	83%	5.64%	3.88%	-3.52%	16.12%

This table summarize depicts L1 and L2 ICAPM strategies for all ratings. Each of the rows in the panel capture risk partitions of excess liquidity (XS), liquidity (Lq) and default (Def). The columns provide the counts of those strategies with non-negative returns (>=0), negative returns (<0), number of significant, insignificant, and all. Then the percentage of non-negative with significant alpha strategies are calculated for the risk partition as well as the average, median, minimum (min), and maximum (max) returns. Panel A summarizes for the L2 strategies. Panel B summarizes for the L1 1010 strategies. Panel C summarizes for the L1 5050 strategies.

Finally we compress the ICAPM significant trading strategies across all ratings in Table 22. There again we observe the L2 strategies outperforming the L1 1010 and L1 5050 strategies in excess liquidity and liquidity strategies compared with default. Across all ratings 100% of significant excess liquidity and liquidity strategies for L2 also exhibited positive returns ranging from 9.09% to 41.37% for excess liquidity and 14.34% to 39.95% for liquidity strategies. In contrast, the default strategies for L2 were positive in only 33% of the cases ranging from -34.67% to 17.08%. In the case of the L1 strategies, the 1010 decile strategies performed somewhat comparatively worse than the 5050 strategies overall with 33% to 50% of the strategies positive and significant for the 1010 decile strategies compared with 83% for the 5050. The returns statistics for 5050 strategies compared with 1010 strategies were generally better overall.

CMBX liquidity and excess liquidity risk pricing by definition, as discussed in Christopoulos (2017) and Christopoulos and Jarrow (2018), provide insights into sentiments of CMBX pricing risks apart from idiosyncratic risks associated with CMBX collateral pools. This is because CMBX risk pricing of liquidity and excess liquidity are residual measures apart from explicitly modelled risks of default, loss and interest rate volatility at the loan level, aggregated to the bond level using fair value techniques. Liquidity and excess liquidity risk pricing provides insights into investor sentiment of risk apart from rate volatility and default experiences as captured in the loan level historical state transitions as discussed in Christopoulos and Barratt (2016). In this sense, at the securities level following fair value aggregation, the liquidity and excess liquidity measures articulate investor sentiment of real estate securities more broadly than the historical analysis of default and rates and their relationship in current CMBX collateral vis a vis simulation. As such, the positive and significant performance for REIT trading strategies using excess liquidity and liquidity CMBX measures makes sense, particularly for the L2 strategies.

As shown in the cross-sectional analysis, we find considerably greater volatility in risk partition pricing at the start of each trading day for all risk partitions compared to the end of each trading day, which contrast with the much more muted volatility for REITs from the start to end of the trading day. This suggests that the monthly and daily analysis supporting insights provided by the theoretical liquidity risk partitions persist at the intraday level (our *third main result*). The success of the trade strategies and their significance confirm that the insights of CMBX risk partitions bring valuable insights to the related REIT sector, our *fourth main result*. This expands the application of CMBX risk partitions, intraday, to a broader array of commercial real estate securities. Together these main and supporting results completes the validation of the Intraday Model.

5 Summary

All markets move, but some sectors move more rapidly than others. This is true with respect to observable differences in the frequency and volume of trading across products. It is also true with respect to observable differences in the rate of adoption of new technology and the characteristics of such technology. While it may be true that prices of individual buildings within the real estate asset class may change less frequently than interest rates, the same cannot be said of CMBS, CMBX and REITs. Equity and fixed income derivatives within capital markets change contemporaneously with changes in interest rates and risk premia. Despite their size and importance in capital markets, near real time information content for credit sensitive US fixed income has only recently come into focus in the microstructure literature. CMBX is one segment of the fixed income market where sparsity of meaningful real time data obscures insights into underlying risk pricing effects of the underlying asset class. Because of that, there is value in understanding the intraday risk composition of the $700 billion CMBS sector and its CMBX derivatives. In this first study into CMBX microstructure we provide such intraday insights into CMBX liquidity with crossover insights into the related REIT sector before and during the Covid-19 pandemic.

In the two earlier studies of Christopoulos (2017) and Christopoulos and Jarrow (2018) focusing on CMBX risk partitioning, the frequency of capture is monthly. This reflected the limited availability of capital resources for computation and limited frequency of available data. Because the results in those studies were quite good, we were able to leverage them and increase the frequency of risk partition estimation to daily and then intraday pacing in this study. By addressing some of the gaps in the frequency and accuracy of risk information in the CMBX market, we advance the literature in line with more developed markets.

By estimating millions of intraday CMBX risk decompositions for all investment grade credits in 15 second intervals we reveal apparent regular volatility pockets of risk partition pricing in the cross-section over the sample period. These cross-sectional characteristics reveal new insights into risk measure price sensitivity to changing market conditions. We investigate into these phenomena in the related REIT sector. By fusing the CMBX risk partition signals with REIT pricing in 54 day trading strategies, we were able to achieve significant ICAPM $\alpha's$ in about 96% of the strategies and positive cumulative returns ranging from 0.73% to 48.74% in about 65% of the strategies during the Covid pandemic. By our results, the fusion of CMBX risk partition signals and REIT pricing, appears to capture a broader investor sentiment with respect to the pricing of real estate securities apart from the idiosyncratic risks of default and rate risks in specific collateral pools underlying CMBX. If more broad real estate risk sentiments of investors are embedded within the CMBX residual risk partitions of liquidity and excess liquidity, it would make sense if those measures revealed pricing insights into real estate risk in securities apart from CMBX. This is certainly the case in the assessment of the relative value of REITs with CMBX risk partitions in this study.

As such, this paper appears to disclose a crossover liquidity sentiment with respect to commercial real estate securities and their pricing in the mind of the market, separate from the idiosyncratic risks of specific collateral underlying REIT and CMBX securities. To our knowledge, this is the first study of its kind to investigate projections of reduced form CMBX risk decompositions on a daily and intraday basis. Since the pace of our intraday risk partitions exceeds the typical pace of CMBX trading, our results validate the theoretical insights independent of market consensus with respect to both REITs and CMBX. In this sense, our approach represents a next step in the evolution of microstructure crossover insights between credit sensitive securitized sectors, such as CMBX, and more mature sectors such as REITs.

This study begs the question as to 'What insights would the direct simulation approach of Christopoulos (2017) and Christopoulos and Jarrow (2018) provide intraday?' compared with the approach we introduce in this paper. The answer to that question, and others, depends upon the availability of highly costly comprehensive data at loan, bond and deal levels and expensive high performance computing resources, and is thus left to future research. Additionally, considering optimized trading strategies intraday, and tactically across risk partition signals, would also be interesting. Given the rapid pacing of interest rates, such extensions with interest rate risk pricing partitions in conjunction with REITs would build on the extensive prior work in the literature related to interest rate term structure models. Also, establishing further connections between intraday risk partition pricing and CMBX actual pricing would be exciting, and that line of inquiry is purely an issue of data accessibility. Finally, from a policy perspective, investigating into the longer term relationship (if any) between intraday signals of risk partitions and manifestation of actual hazards in the real estate asset class would be a interesting area for exploration. Real estate lending criteria, risk retention, and capital constraints all have links to real estate capital markets. Research inquiries into the nature of those links, with the intraday approach we have introduced in this paper, before hazards manifest, seems a natural next level of inquiry. These and other questions are left to future work.

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A Online Appendix

A.1 CMBX microstructure fundamentals

Figure 11 as of the close of business 3/18/2020 at 4:15pm EST was generously provided by one of the world's leading money managers. CMBX does not trade on a price basis, rather in keeping with most other credit sensitive fixed income securities, CMBX trades on a spread over the relevant risk free rate (in bps). Figure 11 depicts an end-of-day summary for actively traded CMBX Series (6-13).[23]

Figure 11: Closing quotes of bid-ask spreads and 1 day changes to mid-market spreads (3/18/2020)


This Message is Firm-forwardable

INDX	19 CMBX.13	CHG	18 CMBX.12	CHG	17 CMBX.11	CHG	16 CMBX.10	CHG
AAA	144/134	+19	133/123	+19	122/111	+18	112/102	+18
AS	255/205	+51	235/185	+44	220/170	+42	205/165	+37
AA	330/280	+66	305/265	+65	310/260	+72	300/260	+76
A	460/390	+89	430/380	+84	410/360	+83	405/355	+89
BBB-	815/755	+142	790/750	+149	740/690	+117	720/670	+104
BB	1200/1050	+215	1350/1100	+345	1150/1000	+204	1200/1050	+207

INDX	15 CMBX.9	CHG	14 CMBX.8	CHG	13 CMBX.7	CHG	12 CMBX.6	CHG
AAA	103/93	+18	96/86	+18	86/86	+22	78/68	+15
AS	200/150	+43	185/135	+43	170/120	+41	150/120	+30
AA	285/235	+69	280/230	+84	255/205	+78	260/210	+65
A	390/340	+95	415/365	+120	375/325	+88	485/435	+88
BBB-	725/670	+105	810/760	+141	800/750	+140	1395/1265	+233

This screenshot shows a Bloomberg post on 3/18/2020 for the quoted bid-ask spreads for CMBX Series 6 through 13 across AAA through BB tranches for Series 10 through 13 and AAA through BBB- for Series 6 through 15. The bid side is on the left of the '/' and the offered side on the right. Bid and Ask are depicted in basis points (bps). The column 'CHG' indicates the change from the prior end of mark to market mid-market spread, and is also quoted in bps..

For each column the year of issuance (expressed in two digits, so 19 is 2019, 18 is 2018, etc.) of the CMBX Series and the CMBX Series name are provided. For example, in the top left, 19 CMBX.13 refers to CMBX Series 13 issued in 2019 ('19'). Below each Series column heading are the bid-side and the ask-side of the bid-ask spread, s_t, which bound the mid-market risk premium, S_t, for each of the tranches (AAA through Ba) in the corresponding credit rating rows. Immediately to the right under each column is the 'CHG' which is the change in mid-market spread, ΔS_t, day over day at the close, where $\Delta S_t = S_t - S_{t-1}$. These quotes are for fixed-income risk premia from the market-maker's perspective.

In OTC fixed-income markets, pricing (bid-side, ask-side and mid-market) is typically

[23]While this information on bid-ask spreads is posted in Bloomberg from dealers to investors, it is not recorded in a central repository with Markit. This is one motivation for our paper.

quoted in terms of risk premia above the relevant risk-free rate. As described in Fabozzi (2016), for amortizing securities, the relevant risk-free rate is a linearly interpolated benchmark risk-free rate (treasuries or interest rate swaps spreads) corresponding to the weighted average life (WAL) of the security being priced described below, where

$$\mathrm{wal}_{jk}(t) = \begin{cases} \dfrac{\left(\frac{\sum_{t=s}^{T} t\hat{\mathbb{A}}_{jk}(t)}{\sum_{t=s}^{T} \hat{\mathbb{A}}_{jk}(t)}\right)}{12}, & \text{for } \sum_{t=1}^{T} F_{jk}(t) > 0, \text{ and} \\ 0, & \text{otherwise} \end{cases} \tag{35}$$

is the weighted average time (in years) to the receipt of the promised monthly principal cashflows, $F_{jk}(t)$, received at month t, of the k-th bond/rating cohort in the j-th CMBX Series.

Example 1. For end-of-day 3/18/2020, the market-maker's bid-side, B_t, for CMBX 13 AAA is 144 bps over the risk free rate, while the market maker's ask-side, A_t, is 134 bps over the risk free rate. Thus total bid-ask spread, s_t is the difference between the bid-side and the offer-side, $s_t = B_t - A_t$ which in this example 10 bps (144-134). The mid-market spread for 3/18/2020, S_t, is 139 bps which is the midpoint between the bid-side and the offer side, $S_t = \frac{(B_t+A_t)}{2}$. Finally from the change in mid market spread column for each Series, we note that S_t is 19 bps wider than S_{t-1} the prior day's (3/17/2020) close, and so $S_{t-1} = S_t - 19 = 120$ bps for this example.

In OTC fixed-income markets, bid-ask spreads are sometimes expressed in conjunction with 'sizes' (quoted face amounts in currency) though they are not always recorded.[24] Such sizes for which the bid-ask spread quotes are valid, indicate the 'depth' of the quote and, in the aggregate, the depth of the market (or sector) overall. Quotes of the bid and ask risk premia (and corresponding sizes) are temporal. Thus, even in the absence of requests for execution, the bid-ask spread between bid-side and ask-side risk premia (spreads), and corresponding sizes, may change reflecting the market-maker's perception of risk due to a variety of factors including changing market conditions and inventories. Bid-ask spreads equal to zero are referred to as 'locked-markets' where the market-maker is willing to buy and sell at the same spread, such that the bid-side and ask-side of the bid-ask spread are equal to one another.

[24] As for example in our time series which only includes spreads, and as shown in the post in Figure 11.

CMBX trading frequency had at one point been roughly 10x the frequency observed in the cash CMBS market as noted by SIFMA. The recording of trading in CRE derivatives is increasing the flow of information as noted in Hollified, Nekyudov and Spatt (2017). Nevertheless, during Covid and the concomitant deterioration in CRE, CMBX has too deteriorated in terms of its actual (not-modelled) liquidity. Information digestion is wide and trading execution directly in the screens make such digestion immediate. When execution is done dealer to client, the digestion of information is much longer as only the market making desk that executes would have "real time" info.

Interestingly, despite some advances in information flow, the CMBX market still does not provide reliable intraday recorded bid-ask spreads, mid-market spreads, corresponding trade execution prices, transaction volumes, or, importantly, risk decomposition monitoring in real time. As such, the information content available for CMBX is still limited and is consistent with the earlier period in the literature in which Roll (1984) and Thompson and Waller (1987) introduce their models. Those models and their adaptations are thus appropriate for CMBX daily bid-ask estimation and provide insight in conjunction with our increased frequency estimation of indexed risk decomposition.

A.2 Derivation of the Absolute Roll Measure (Christopoulos (2020))

In this section I introduce a novel adaptation of Roll (1984) using imaginary numbers.

Claim. I claim the use of imaginary numbers within the radicand of Eq. (1) allows a new measure for the effective bid-ask spread estimate based on Roll (1984) referred to as the Absolute Roll Measure, and defined as:

$$\hat{s} = 2\sqrt{\left|-\mathrm{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|} \tag{36}$$

Definition. Following Roll (1984), let the autocovariance of asset returns, s, be defined[25] such that

$$s = \begin{cases} s^{+} = 2\sqrt{-\mathrm{COV}(\Delta P_t, \Delta P_{t+1})} & \text{for } \mathrm{COV}(\Delta P_t, \Delta P_{t+1}) > 0 \\ s^{-} = 2\sqrt{-\mathrm{COV}(\Delta P_t, \Delta P_{t+1})} & \text{for } \mathrm{COV}(\Delta P_t, \Delta P_{t+1}) < 0 \\ s^{z} = 2\sqrt{-\mathrm{COV}(\Delta P_t, \Delta P_{t+1})} & \text{for } \mathrm{COV}(\Delta P_t, \Delta P_{t+1}) = 0 \end{cases} \tag{37}$$

with the imaginary number, i, defined[26] as

$$i = \sqrt{-1} \iff i^2 = -1 \tag{38}$$

Conjecture. For s^{+},

$$\begin{aligned} s^{+} =& 2\sqrt{\text{-}\mathrm{COV}(\Delta P_t, \Delta P_{t+1})} \\ =& 2\sqrt{\text{-}1 \times \underbrace{\mathrm{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{>0}} \\ =& 2\underbrace{\sqrt{-1}}_{=i}\sqrt{\underbrace{\mathrm{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{>0}} \\ =& i \times \underbrace{2\sqrt{\mathrm{COV}(\Delta P_t, \Delta P_{t+1})}}_{=y} \end{aligned}$$

Let $x = 0$ and $y = 2\sqrt{\underbrace{\mathrm{COV}(\Delta P_t, \Delta P_{t+1})}_{>0}}$, so by complex numbers

$$s^{+} = yi$$

with the complex conjugate of s^{+} defined as

$$\overline{s^{+}} = -\,yi$$

[25] This is motivated by Harris (1990) to 'preserve the sign' of the autocovariance, but carries different signs for s^{+} inside and outside the radical.

[26] See Simon and Blume (1994).

and their product

$$\begin{aligned} s^+\overline{s^+} &= -y^2\underbrace{i^2}_{=-1} \\ &= -y^2 \times -1 \\ &= y^2 \end{aligned}$$

Taking the square root of both sides

$$\begin{aligned} \underbrace{\sqrt{s^+\overline{s^+}}}_{=|s^+|} &= \sqrt{y^2} \\ \left|s^+\right| &= y \\ &= 2\sqrt{\underbrace{\text{COV}(\Delta P_t, \Delta P_{t+1})}_{>0}} \end{aligned}$$

$$\therefore \left|s^+\right| = 2\sqrt{\underbrace{\text{COV}(\Delta P_t, \Delta P_{t+1})}_{>0}} \tag{39}$$

which is the distance of s^+ from the origin, in the complex plane.

Similarly, for s^-,

$$\begin{aligned} s^- &= 2\sqrt{\text{-COV}(\Delta P_t, \Delta P_{t+1})} \\ &= 2\sqrt{\text{-1}\times\underbrace{\text{COV}(\Delta P_t, \Delta P_{t+1})}_{<0}} \\ &= 2\sqrt{\text{-1}\times\underbrace{-1 \times \text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{<0}} \\ &= 2\sqrt{\underbrace{-1\times -1}_{=1} \times \underbrace{\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{>0}} \\ &= 2\sqrt{1\times\underbrace{\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{>0}} \end{aligned}$$

$$= 2\sqrt{\underbrace{\text{COV}(\Delta P_t, \Delta P_{t+1})}_{>0}} = \left|s^-\right| \tag{40}$$

by absolute values, which is the distance of the real number, s^- from the origin.

Finally, for s^z,

$$
\begin{aligned}
s^z =& 2\sqrt{\text{-COV}(\Delta P_t, \Delta P_{t+1})}\\
=& 2\sqrt{\text{-1}\times\underbrace{\text{COV}(\Delta P_t, \Delta P_{t+1})}_{=0}}\\
=& 2\sqrt{\text{-1}\times\underbrace{-1\times\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{=0}}\\
=& 2\sqrt{\underbrace{-1\times-1}_{=1}\times\underbrace{\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{=0}}\\
=& 2\sqrt{1\times\underbrace{\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)}_{=0}}
\end{aligned}
$$

$$
=2\sqrt{\underbrace{\text{COV}(\Delta P_t, \Delta P_{t+1})}_{=0}} = |s^z| \tag{41}
$$

by absolute values, which is the distance of the real number, s^z from the origin, at the origin, which is zero.

Solution. Eqs. (39 and 40) yield the result that $|s^+| = |s^-|$. The radicands for both Eqs. (39 and 40) are shown to be $\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right) > 0$. Post the evaluation with imaginary numbers above and on the domain partitions in Eq. (37), for each of those radicands $\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right) = \left|\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|$ allowing for substitution of $\left|\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|$ for $\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)$ in the radicands for Eqs. (39, 40 and 41) allowing them to be restated as:

$$
\left|s^+\right| = 2\sqrt{\left|\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|}\text{for COV}(\Delta P_t, \Delta P_{t+1}) > 0 \tag{42}
$$

$$
\left|s^-\right| = 2\sqrt{\left|\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|}\text{for COV}(\Delta P_t, \Delta P_{t+1}) < 0 \tag{43}
$$

$$
\left|s^z\right| = 2\sqrt{\left|\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|}\text{for COV}(\Delta P_t, \Delta P_{t+1}) = 0 \tag{44}
$$

Since Eqs. (42, 43, and 44) yield the same value, it follows that

$$
\left|s^+\right| = \left|s^-\right| = \left|s^z\right| = 2\sqrt{\left|\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|} \text{ for } -\infty < \text{COV}\left(\Delta P_t, \Delta P_{t+1}\right) < \infty \tag{45}
$$

But

$$2\sqrt{\left|\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|} = 2\sqrt{\left|-\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|} \text{ for } -\infty < \text{COV}\left(\Delta P_t, \Delta P_{t+1}\right) < \infty \tag{46}$$

Therefore,

$$|s| = 2\sqrt{\underbrace{\left|-\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)\right|}_{\geq 0}} = s \tag{47}$$

The absolute value of a non-negative number is simply the number itself. Since the radicand in Eq. (47) is strictly non-negative, Eq. (47) equals Eq. (36), and thus $|s| = s = \hat{s}$ for *all* $\text{COV}\left(\Delta P_t, \Delta P_{t+1}\right)$ through the use of imaginary numbers. As such, $\hat{s}$ is the *Absolute Roll Measure*, an adaptation of the effective spread of Eq. (1) with imaginary numbers. ■

This Absolute Roll Measure, $\hat{s}$, guarantees a strictly non-negative bid-ask spread observable for all price changes in all traded asset markets. The Absolute Roll Measure, $\hat{s}$, is the magnitude of s, regardless of whether s is real or imaginary. For observations where $\text{COV}(\Delta P_t, \Delta P_{t+1}) > 0$, the interpretation is the corresponding effective bid-ask spread exists in the complex plane.

A.3 VAR Tables

Table 23: VAR AAA CMBX

Panel A: CJ Liquidity_AAA VAR with Absolute Roll

	Estimate	StdError	Pr(>|t|)	Sig
ABSRollAAA.L1	0.57265	0.01881	<2.00E-16	***
CJlq.L1	3.46923	1.10395	0.001692	**
ABSRollAAA.L2	0.02048	0.0217	0.34554	
CJlq.L2	0.17021	1.17189	0.884529	
ABSRollAAA.L3	0.36228	0.0217	<2.00E-16	***
CJlq.L3	1.88855	1.17416	0.107854	
ABSRollAAA.L4	0.07374	0.02237	0.000992	***
CJlq.L4	0.86971	1.17343	0.458652	
ABSRollAAA.L5	-0.23068	0.02237	<2.00E-16	***
CJlq.L5	-2.03651	1.17257	0.082533	.
ABSRollAAA.L6	0.04481	0.02177	0.039601	*
CJlq.L6	-1.32114	1.17077	0.259232	
ABSRollAAA.L7	-0.02312	0.02178	0.288464	
CJlq.L7	0.36954	1.16785	0.751703	
ABSRollAAA.L8	0.08763	0.01883	3.40E-06	***
CJlq.L8	-1.24547	1.1027	0.258794	
			0.863177	
const	-0.07769	0.45078		
F-stat p-value	<2.2e-16			
Adj. Rsq	0.7307			
df	2803			

Panel B: CJ Excess Liquidity_AAA VAR with Absolute Roll

	Estimate	StdError	Pr(>|t|)	Sig
ABSRollAAA.L1	0.569389	0.018874	<2.00E-16	***
CJxslq.L1	-1.417582	0.816753	0.082739	.
ABSRollAAA.L2	0.025181	0.021722	0.246456	
CJxslq.L2	0.391611	0.887517	0.659071	
ABSRollAAA.L3	0.356316	0.02165	<2.00E-16	***
CJxslq.L3	-1.876339	0.890014	0.035101	*
ABSRollAAA.L4	0.062477	0.022652	0.005852	**
CJxslq.L4	0.821246	0.887263	0.354736	
ABSRollAAA.L5	-0.24152	0.02266	<2.00E-16	***
CJxslq.L5	2.559536	0.888143	0.003983	**
ABSRollAAA.L6	0.052903	0.022629	0.019467	*
CJxslq.L6	0.061837	0.889057	0.944554	
ABSRollAAA.L7	-0.009097	0.022607	0.687434	
CJxslq.L7	-3.042457	0.886945	0.000612	***
ABSRollAAA.L8	0.093711	0.021572	1.45E-05	***
CJxslq.L8	-0.04969	0.889555	0.955458	
ABSRollAAA.L9	0.011963	0.021642	0.580472	
CJxslq.L9	0.655737	0.88623	0.459413	
ABSRollAAA.L10	-0.055848	0.018825	0.003036	**
CJxslq.L10	-1.75708	0.814223	0.031014	*
const	1.791993	0.291729	9.27E-10	***
F-stat p-value	<2.2e-16			
Adj. Rsq	0.7318			
df	2797			

Panel C: CJ Liquidity_AAA VAR with Thompson Waller

	Estimate	StdError	Pr(>|t|)	Sig
TWAAA.L1	0.937575	0.018911	<2.00E-16	***
CJlq.L1	1.923353	0.690751	0.005398	**
TWAAA.L2	-0.263708	0.025932	<2.00E-16	***
CJlq.L2	-1.330626	0.731897	0.069163	.
TWAAA.L3	0.097619	0.026325	0.000213	***
CJlq.L3	-2.113251	0.733776	0.004007	**
TWAAA.L4	-0.011663	0.026395	0.658618	
CJlq.L4	2.269999	0.733637	0.001993	**
TWAAA.L5	0.041152	0.026373	0.118773	
CJlq.L5	-2.319467	0.735903	0.001639	**
TWAAA.L6	0.043547	0.026321	0.098144	.
CJlq.L6	1.580418	0.73711	0.032113	*
TWAAA.L7	0.004757	0.026276	0.856347	
CJlq.L7	0.046257	0.735759	0.949875	
TWAAA.L8	0.120366	0.026173	4.44E-06	***
CJlq.L8	-0.280125	0.735551	0.703353	
TWAAA.L9	-0.032026	0.025815	0.214853	
CJlq.L9	0.558292	0.733437	0.446602	
TWAAA.L10	-0.057352	0.018837	0.002352	**
CJlq.L10	0.296099	0.691674	0.668618	
const	0.143671	0.296622	0.628169	
F-stat p-value	<2.2e-16			
Adj. Rsq	0.7271			
df	2797			

Panel D: CJ Excess Liquidity_AAA VAR with Thompson Waller

	Estimate	StdError	Pr(>|t|)	Sig
TWAAA.L1	0.9254143	0.0188643	<2.00E-16	***
CJxslq.L1	-0.7926893	0.510231	0.120396	
TWAAA.L2	-0.2539046	0.0257194	<2.00E-16	***
CJxslq.L2	1.2122908	0.5551738	0.029073	*
TWAAA.L3	0.0950697	0.0260432	0.000267	***
CJxslq.L3	1.053332	0.5572731	0.058841	.
TWAAA.L4	-0.0191766	0.0261038	0.462628	
CJxslq.L4	-0.760877	0.5551548	0.17062	
TWAAA.L5	0.031985	0.0260955	0.220419	
CJxslq.L5	-1.4215787	0.5558235	0.010592	*
TWAAA.L6	0.0498144	0.0260796	0.056224	.
CJxslq.L6	-0.3272787	0.5563596	0.556412	
TWAAA.L7	0.0002499	0.0260945	0.99236	
CJxslq.L7	-0.0756219	0.5549253	0.891614	
TWAAA.L8	0.1238949	0.0260181	2.02E-06	***
CJxslq.L8	0.7700658	0.5554689	0.165755	
TWAAA.L9	-0.0341029	0.0256475	0.183732	
CJxslq.L9	-1.0670362	0.5534818	0.053973	.
TWAAA.L10	-0.0691792	0.0188248	0.000242	***
CJxslq.L10	-0.591579	0.5086069	0.244874	
const	0.9798136	0.1642095	2.72E-09	***
F-stat p-value	<2.2e-16			
Adj. Rsq	0.7266			
df	2797			

This table shows the results for the VAR with the daily AAA CMBX bid-ask estimates as the endogenous variables and liquidity (CJ Liquidity) and excess liquidity (CJ xslq) as the exogenous variables. The lags, L#, are determined by the Akaike Information Criteria (AIC) to a maximum of 10 lags. Estimates are provided with standard errors, probabilities and statistical significance. All panels capture the lagged effective bid-ask spread for the labelled model. Panels A and B report the estimates using the Absolute Roll measure of Christopoulos (2020) for AAA effective bid-ask spreads with the lagged liquidity (CJlq, Panel A) and excess liquidity (CJxslq, Panel B). Panels C and D report the estimates using the effective bid-ask measure of Thompson and Waller (1987)for AAA effective bid-ask spreads with the lagged liquidity (CJlq, Panel C) and excess liquidity (CJxslq, Panel D). The F-test p-value and the value of the VAR's Adjusted R-squared and degrees of freedom are reported at the bottom of each panel. ***/**/*/' correspond to 0.1%, 1%, 5% and 10% levels of significance.

Table 24: VAR BBB CMBX

Panel A: CJ Liquidity__BBB VAR with Absolute Roll

	Estimate	StdError	Pr(>|t|)	Sig
ABSRollBBB.L1	0.98165	0.01869	<2.00E-16	***
CJlq.L1	5.238883	9.400263	0.5774	
ABSRollBBB.L2	-0.108892	0.026028	2.96E-05	***
CJlq.L2	2.002425	9.96197	0.8407	
ABSRollBBB.L3	0.066317	0.0261	0.0111	*
CJlq.L3	12.806762	9.980923	0.1996	
ABSRollBBB.L4	-0.525612	0.02613	<2.00E-16	***
CJlq.L4	-7.447551	9.974604	0.4553	
ABSRollBBB.L5	0.420097	0.026801	<2.00E-16	***
CJlq.L5	2.422779	9.99648	0.8085	
ABSRollBBB.L6	-0.008049	0.026127	0.758	
CJlq.L6	6.362776	9.972082	0.5235	
ABSRollBBB.L7	0.035015	0.026087	0.1796	
CJlq.L7	3.355688	9.976895	0.7366	
ABSRollBBB.L8	-0.213281	0.026012	3.64E-16	***
CJlq.L8	-1.770599	9.954828	0.8588	
ABSRollBBB.L9	0.147624	0.01868	3.89E-15	***
CJlq.L9	5.24658	9.3903	0.5764	
const	-1.905478	3.957207	0.6302	
F-stat p-value	<2.2E-16			
Adj. Rsq	0.7105			
df	2800			

Panel B: CJ Excess Liquidity__BBB VAR with Absolute Roll

	Estimate	StdError	Pr(>|t|)	Sig
ABSRollBBB.L1	0.974638	0.018902	<2.00E-16	***
CJxslq.L1	3.808602	6.963996	0.5845	
ABSRollBBB.L2	-0.108012	0.026256	4.01E-05	***
CJxslq.L2	-9.990817	7.591396	0.1883	
ABSRollBBB.L3	0.06384	0.026026	0.0142	*
CJxslq.L3	-5.195364	7.617645	0.4953	
ABSRollBBB.L4	-0.526175	0.026044	<2.00E-16	***
CJxslq.L4	4.416655	7.585277	0.5604	
ABSRollBBB.L5	0.417218	0.027874	<2.00E-16	***
CJxslq.L5	0.896584	7.594219	0.906	
ABSRollBBB.L6	-0.009913	0.027874	0.7221	
CJxslq.L6	-8.464498	7.592601	0.265	
ABSRollBBB.L7	0.033203	0.02605	0.2026	
CJxslq.L7	-7.166614	7.575375	0.3442	
ABSRollBBB.L8	-0.213458	0.026026	3.57E-16	***
CJxslq.L8	3.646027	7.582864	0.6307	
ABSRollBBB.L9	0.147282	0.02625	2.21E-08	***
CJxslq.L9	0.88437	7.555367	0.9068	
ABSRollBBB.L10	-0.006246	0.018888	0.7409	
CJxslq.L10	-9.164495	6.932413	0.1863	
const	13.769115	2.034755	1.60E-11	***
F-stat p-value	<2.2E-16			
Adj. Rsq	0.7126			
df	2797			

Panel C: CJ Liquidity__BBB VAR with Thompson Waller

	Estimate	StdError	Pr(>|t|)	Sig
TWBBB.L1	1.0877	0.01888	<2.00E-16	***
CJlq.L1	-5.92985	19.62361	0.762538	
TWBBB.L2	-0.15956	0.02786	1.13E-08	***
CJlq.L2	-3.42982	20.8086	0.869092	
TWBBB.L3	-0.1258	0.02799	7.28E-06	***
CJlq.L3	13.72369	20.85119	0.510481	
TWBBB.L4	0.18595	0.02808	4.20E-11	***
CJlq.L4	10.90645	20.83589	0.600706	
TWBBB.L5	-0.31908	0.02794	<2.00E-16	***
CJlq.L5	-12.92307	20.86593	0.535744	
TWBBB.L6	0.23683	0.02794	<2.00E-16	***
CJlq.L6	18.37461	20.86065	0.378488	
TWBBB.L7	0.05064	0.02807	0.07134	.
CJlq.L7	10.14835	20.8232	0.626043	
TWBBB.L8	-0.07936	0.02799	0.00461	**
CJlq.L8	-16.04993	20.82542	0.440956	
TWBBB.L9	0.0961	0.02787	0.000572	***
CJlq.L9	53.52448	20.76675	0.010005	*
TWBBB.L10	-0.0609	0.01887	0.001263	**
CJlq.L10	-7.09095	19.60909	0.717667	
const	-7.95504	8.46828	0.347611	
F-stat p-value	<2.2E-16			
Adj. Rsq	0.8436			
df	2797			

Panel D: CJ Excess Liquidity__BBB VAR with Thompson Waller

	Estimate	StdError	Pr(>|t|)	Sig
TWBBB.L1	1.07463	0.01886	<2.00E-16	***
CJxslq.L1	-18.46408	14.48756	0.2026	
TWBBB.L2	-0.15629	0.02766	1.75E-08	***
CJxslq.L2	13.63989	15.77238	0.387225	
TWBBB.L3	-0.12526	0.02776	6.70E-06	***
CJxslq.L3	-13.66792	15.82018	0.387687	
TWBBB.L4	0.17938	0.02785	1.40E-10	***
CJxslq.L4	-25.79096	15.75685	0.101783	
TWBBB.L5	-0.32091	0.0277	<2.00E-16	***
CJxslq.L5	-11.01383	15.77891	0.485229	
TWBBB.L6	0.23369	0.0277	<2.00E-16	***
CJxslq.L6	14.18664	15.77523	0.368571	
TWBBB.L7	0.04891	0.02784	0.079033	.
CJxslq.L7	-7.14949	15.74004	0.649704	
TWBBB.L8	-0.08079	0.02775	0.003633	**
CJxslq.L8	9.99069	15.75457	0.526037	
TWBBB.L9	0.09769	0.02763	0.000413	***
CJxslq.L9	-30.32073	15.7029	0.053596	.
TWBBB.L10	-0.07402	0.01883	8.69E-05	***
CJxslq.L10	-13.48859	14.43483	0.350154	
const	35.39276	4.86716	4.58E-13	***
F-stat p-value	<2.2E-16			
Adj. Rsq	0.8456			
df	2797			

This table shows the results for the VAR with the daily BBB CMBX bid-ask estimates as the endogenous variables and liquidity (CJ Liquidity) and excess liquidity (CJ xslq) as the exogenous variables. The lags, L#, are determined by the Akaike Information Criteria (AIC) to a maximum of 10 lags. Estimates are provided with standard errors, probabilities and statistical significance. All panels capture the lagged effective bid-ask spread for the labeled model. Panels A and B report the estimates using the Absolute Roll measure of Christopoulos (2020) for BBB effective bid-ask spreads with the lagged liquidity (CJlq, Panel A) and excess liquidity (CJxslq, Panel B). Panels C and D report the estimates using the effective bid-ask measure of Thompson and Waller (1987)for BBB effective bid-ask spreads with the lagged liquidity (CJlq, Panel C) and excess liquidity (CJxslq, Panel D). The F-test p-value and the value of the VAR's Adjusted R-squared and degrees of freedom are reported at the bottom of each panel. ***/**/*/' correspond to 0.1%, 1%, 5% and 10% levels of significance.

A.4 Proper specification of the Daily Model

One might consider whether the Daily Model has some collinearity in its terms. To address this, consider the first three terms of the model from Section 3:

$$v_t \equiv [f_t - f_{t-1}]^2 = f_t^2 - 2f_t f_{t-1} + f_{t-1}^2 \tag{48}$$

$$v_{t-1} \equiv [f_{t-1} - f_{t-2}]^2 = f_{t-1}^2 - 2f_{t-1} f_{t-2} + f_{t-2}^2 \tag{49}$$

$$[v_t - v_{t-1}]^2 = v_t^2 - 2v_t v_{t-1} + v_{t-1}^2 \tag{50}$$

We know that for two linearly independent terms, their squared difference is also linearly independent of the former two. Substituting the right-hand terms of the equation for v_t, Eq. (48), and for v_{t-1}, Eq. (49), into the right-hand side terms for Eq. (50), we get the following expansion for the PCA factors.

$$\begin{aligned}
= \quad & (f_t^2 - 2f_t f_{t-1} + f_{t-1}^2)^2 - 2(f_t^2 - 2f_t f_{t-1} + f_{t-1}^2)(f_{t-1}^2 - 2f_{t-1} f_{t-2} + f_{t-2}^2) + (f_{t-1}^2 - 2f_{t-1} f_{t-2} + f_{t-2}^2)^2 \\
= \quad & (f_t^4 - 4f_t^3 f_{t-1} + 6f_t^2 f_{t-1}^2 - 4f_t f_{t-1}^3 + f_{t-1}^4) - \\
& (2f_t^2 f_{t-1}^2 - 4f_t^2 f_{t-1} f_{t-2} + 2f_t^2 f_{t-2}^2 - 4f_t f_{t-1}^3 + 8f_t f_{t-1}^2 f_{t-2} - 4f_t f_{t-1} f_{t-2}^2 + 2f_{t-1}^4 - 4f_{t-1}^3 f_{t-2} + 2f_{t-1}^2 f_{t-2}^2) + \\
& (f_{t-1}^4 - 4f_{t-1}^3 f_{t-2} + 6f_{t-1}^2 f_{t-2}^2 - 4f_{t-1} f_{t-2}^3 + f_{t-2}^4) \\
= \quad & (f_t^4 \text{-} 4f_t^3 f_{t-1} + 6f_t^2 f_{t-1}^2 \text{-} 4f_t f_{t-1}^3 + f_{t-1}^4) \text{-} \\
& (2f_t^2 f_{t-1}^2 \text{-} 4f_t^2 f_{t-1} f_{t-2} + 2f_t^2 f_{t-2}^2 \text{-} 4f_t f_{t-1}^3 + 8f_t f_{t-1}^2 f_{t-2} \text{-} 4f_t f_{t-1} f_{t-2}^2 + 2f_{t-1}^4 \text{-} 4f_{t-1}^3 f_{t-2} + 2f_{t-1}^2 f_{t-2}^2) + \\
& (f_{t-1}^4 - 4f_{t-1}^3 f_{t-2} + 6f_{t-1}^2 f_{t-2}^2 - 4f_{t-1} f_{t-2}^3 + f_{t-2}^4) \\
= \quad & f_t^4 - 4f_t^3 f_{t-1} + 4f_t^2 f_{t-1}^2 - 2f_t^2 f_{t-2}^2 + 4f_{t-1}^2 f_{t-2}^2 - 4f_{t-1} f_{t-2}^3 + 4f_t^2 f_{t-1} f_{t-2} - 8f_t f_{t-1}^2 f_{t-2} + 4f_t f_{t-1} f_{t-2}^2 + f_{t-2}^4
\end{aligned} \tag{51}$$

The model is well specified if all the terms of the model are non-collinear. All the terms in Eqs. (48, 49, 50 and 51) are linearly independent, and therefore these first three terms of the Daily Model are non-collinear. Additionally, the fourth term of the Daily Model (the market spread) is a direct empirical observation and therefore independent of the first three. Since all four terms of the Daily Model are non-collinear, the Daily Model is well specified.■