YESHIVA UNIVERSITY

YESHIVA COLLEGE

Semester: Spring 2004

Course: 1520 Advanced Statistical Mechanics
Hours: Monday – Wednesday 6:45 – 8:00

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Assignment 1: Statistics

1) The birthday problem:

a) Calculate the probability that two or more people have the same birthday in a group of N randomly selected people
b) Calculate how many people are needed to make the probability found in (a) just greater than 50%.
c) In a class of 100 people, calculate the probability that all have different birthdays.

2) Poker problems:

a) What is the probability, in a poker hand of 5 cards, of getting a royal flush: 10,J,Q,K,A.
b) What is the probability of getting two pairs (a hand of the type {x,x,y,y,z}).
c) What is the probability of getting a full house ({x,x,x,y,y})

3) A 1000 tickets are sold in a raffle, of which 50 are winning tickets. You purchase 10 tickets. What is the probability that you will get 2 winning tickets?

4) You are a biologist doing a fish tagging survey. You net 100 bass, you tag them and you release them. After waiting long enough for the fish to disperse, a second sample of 100 bass is taken, and you observe that 5 of them are tagged.

a) If the number of bass in the lake is N, what is the probability that a random sample of size 100 will contain 5 tagged fish?
b) How would you estimate the number of bass N in the lake?

5) A dice problem:

I throw a pair of dice until a score of 6 as the sum of the two dice appears for the first time, in which case I stop.

a) What is the likelihood that the game will last exactly seven throws.
b) What is the likelihood that it will last at least seven throws.

6) An old friend:(which can be obtained in a different way, not using generating function ideas).

Consider an harmonic oscillator potential whose levels are (counting from the fundamental up) 0, hw, 2hw, 3hw, 4hw, 5hw,…….
Assume 3 distinguishable (and non interacting) particles in that potential.

a) How many different states are there of energy equal to 3hw? (easy answer 10, why?)
b) Repeat the problem now for 8 distinguishable particles and total energy 17hw.
c) Come up with the general expression for n particles and Mhw total energy.

7) Random walks:

a) Find the probability of getting 270 heads while tossing a fair coin 500 times.

b) Two random walkers start simultaneously to walk in a 1 dimensional in a symmetric random walk. They take steps of the same length and at the same time. What is the probability that after N steps they meet.

8) A man has n similarly looking keys, of which one will open his lock and the others will not.

a) Smart man: What are the chances that he will open the lock in his first attempt, in his first two attempts, in his first three attempts, etc.
b) Silly man (who does not record which key failed and just tries one from the bunch at random again). Same question.

9) An electronic system has a periodic operating cycle of 0.01 seconds. In each of the cycles an event can occur with probability 0.001. What is the probability of observing fewer than 15 events in a 100-second interval?

10) The airline industry problem:

a) A plane has a capacity of 45 passengers. The airline sells 50 tickets. If one person out of 12 is normally a no-show, what is the probability that all the passengers that show up will be accommodated?
b) What is the maximum number of tickets that the airline should sell so that they should be able to accommodate all the show up passengers with probability at least equal to 90%?

11) The neutrino detector:

A certain neutrino detector tries to observe neutrinos from a distant supernova. The observed background in the detector is 1 count/day. After running for two years they observe 3 events in one hour. Is this event rate evidence of a supernova? (Use the Poisson distribution to estimate the probability of obtaining at least this high of an event rate from statistical fluctuations over the course of the experiment.

12) The California Lotto:

A few weeks ago the jackpot was 102 million dollars. A person, paying 1 dollar selects 6 different numbers from 1 to 51. At the drawing, 6 different numbers are randomly selected; if you match these 6 numbers you win.

a) What are the odds of winning with one ticket?
b) For the 102 million dollar jackpot there were three winners. Use this information to estimate the number of players, assuming this number of winners was fairly probable (say bigger than about 10%)